MECHANICAL ENGINEERING

FOR

UPSC Engineering Services Examination, GATE,
State Engineering Service Examination & Public Sector Examination.
(BHEL, NTPC, NHPC, DRDO, SAIL, HAL, BSNL, BPCL, NPCL, etc.)

DESIGN OF MACHINE ELEMENTS

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CHAPTER

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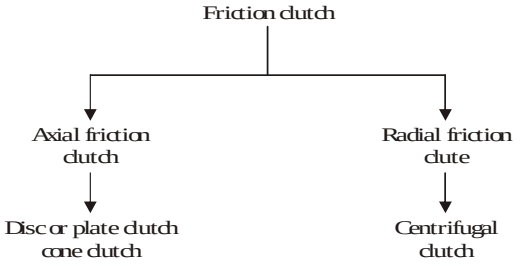
Friction Clutches

A clutch is a machine member used to connect a driving shaft to a driven shaft in order to start and stop driven shaft at will, without stopping the driving shaft.

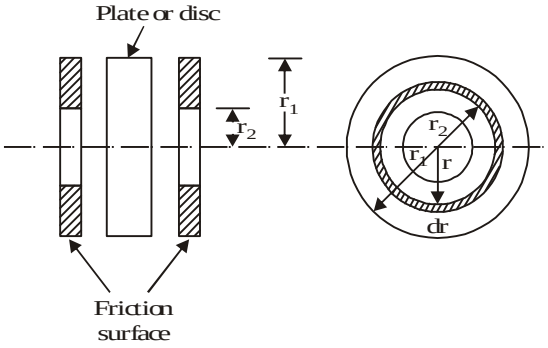
Materials for Friction Clutches

Commonly used materials for friction clutches are

1. **Cast Iron:** It is used dry but operating temperature in limited to 300°C.
2. **Steel:** It is used with oil.
3. **Bronze:** It is used with oil and maximum operating temperature is limited to 150°C.
4. **Powder metal on Cast Iron:** Dry conditions with upto 550°C operating temperatures.



Design of a disc or plate clutch'



Frictional torque transmitted

$$\boxed{T_r = 2\pi M_P r^2 dr}$$

$$F_r = F_{axial} \times \mu$$
$$= P \times 2\pi\, rdr \times \mu$$
$$= 2\pi P \mu r dr$$

$$T_r = F_r \times r$$
$$= 2\pi\rho\mu\, r^2 dr.$$

Here, we have to consider two cases.

1. Uniform Pressure

Where the pressure is uniformly distributed over the entire area of friction face.

$$P = \frac{W}{\pi[r_1^2 - r_2^2]}$$

Total frictional force,

$$T = \int_{r_2}^{r_1} T_r dr = \int_{r_2}^{r_1} 2\pi\mu\rho r^2 dr$$

$$= \frac{2\pi\mu W}{\pi[r_1^2 - r_2^2]}\left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$\boxed{T = \frac{2}{3}\mu w\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

Here mean radius of friction surface,

$$R_m = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

2. Uniform Axial Wear

The basic principal is that the normal wear in all sliding parts is proportional to the work of friction.

Normal wear × work of friction $\propto$ PV

(Normal pressure × Velocity)

or $PV = K$

$$\boxed{P = K/V}$$

The intensity of pressure is inversly proportional to the radial distance i.e.

$P\,r = C$(constant) or

$$\boxed{P = C/r}$$

where, $$\boxed{C = \frac{W}{2\pi(r_1 - r_2)}}$$

So,

$$\boxed{P_{max} = \frac{C}{r_{min}} \quad \text{and} \quad P_{min} = \frac{C}{r_{max}}}$$

Torque, $T_r = 2\pi\mu\rho r^2 dr$

But, $P = C/r$

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$$= 2\pi\mu \frac{C}{r} \times r^2 dr = 2\pi\mu\, crdr$$

Total torque,

$$T = \int_{r_1}^{r_2} T_r dr = \int_{r_1}^{r_2} 2\pi\mu crdr$$

$$T = 2\pi\mu \times C \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = \frac{2\pi\mu \times W}{2\pi(r_1 - r_2)} \frac{[r_1^2 - r_2^2]}{2}$$

$$= \boxed{\frac{1}{2} \times \mu \times W(r_1 + r_2)}$$

Mean radius of friction face,

$$\boxed{R_m = \frac{r_1 + r_2}{2}}$$

- In general the total torque is multiplied with number of frictional surface to get final torque for example a single plate has two frictional surface so n = 2.

$$\boxed{T = n \times \mu \times w \times R_m}$$

- Average pressure in both the cases on frictional face

$$\boxed{P_{aV} = \frac{W}{\pi[r_1^2 - r_2^2]}}$$

- In a new clutch uniform pressure theory is applied, but in an old clutch uniform wear the is applied.
- Uniform pressure theory gives a higher torque as compared to uniform wear theory. Therefore, uniform wear theory should generally be used.

Multiple Disch Clutch

These are used when large torque is to be transmitted. Inner discs (of steel) are fasted to driven shaft, outer disc (bronze) are fastened to housing which is keyed to driving shaft.

Total number of friction surface.

$$\boxed{n = n_1 + n_2 - 1}$$

where, n_1 = No. of dises of driving shaft (bronze).

n_2 = No. of dises of drivien shaft (steel)

Torque transmetted,

$$\boxed{T = n\mu w R_m}$$

Example 1

P = 100 KW; N = 2400 r.p.m. Maximum torque = 500 N; P_{max} = 0.07 N/mm^2 Determine dimensions of clutch and initial compreission of spning, outer radius in 25% more than nera radius.

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Sol. Considering uniform wear

$$T = n + \mu \times w \times R_m$$

$$= \frac{n\mu w(r_1 + r_2)}{2}$$

$$n = 2$$

$$T = 2 \times 0.3 \times w \times \left(\frac{r_1 + r_2}{2}\right)$$

$$C = \frac{w}{2\pi(r_1 - r_2)}$$

also, $P = C/r$

but, $\boxed{r_1 = 1.25\, r_2}$ given

$$P_{max} = \frac{C}{r_{min}} = \frac{C}{r_2}$$

$$C = 0.07 \times r_2$$

$$W = 2\pi \times 0.07\, r_2(1.25\, r_2 - r_2)$$

$$W = 0.1099\, r_2^2$$

$$T = 2 \times \mu \times 0.1099 r_2^2 \frac{(1.25 r_2 + r_2)}{2}$$

$$= 0.3 \times 0.1099 \times 2.55 \times r_2^3$$

$$r_2^3 = \frac{500 \times 10^3}{0.3 \times 2.25 \times 0.1099}$$

$\boxed{r_2 = 189 \text{ mm}}$

$\boxed{r_1 = 1.25\, r_2 = 237 \text{ mm}}$

(b) Initial Compression of Springs

= W/s.

W– Axial force = 0.1099

r_2^2 = 3926 N.

S = total stffness of spmngs = 40 × 8

= 320 N/mm.

Initial compression is $= \frac{3926}{320}$

$\boxed{12.3 \text{ mm}}$

Force on,

Friction surface, = Pressure × Area

$\boxed{P \times \pi[r_1^2 - r_2^2]}$

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when inner and outer radius are consider,

and = $\boxed{P \times 2\pi R_m \times b}$

when mean radius face width are consider,

Where face width,

$$\boxed{b = r_1 - r_2}$$

Example 2

P = 4.5 KW, N = 750 rpm; r_1 = 70 mm; $r_2 = 40$ mm $\mu = 0.1$; P_{av} = 0.35 N/mm^2. Calculate.

1. Total number of steel and bronze plates
2. Actual axial force
3. Actual average pressure
4. Actual maximum pressure.

Sol. $T = n \times \mu \times w \times R_m$

Assuming uniform wear

$$R_m = \frac{r_1 + r_2}{2} = \frac{70 + 40}{2}$$

$$= 55 \text{ mm.}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{4500 \times 10^3 \times 60}{2 \times \pi \times 750}$$

$$= 57296 \text{ N-mm}$$

$$57296 = n \times 0.1 \times \rho_{av} \times \pi \times (r_1^2 - r_2^2) \times 55 \quad \left[P_{av} = \frac{W}{\pi\left[r_1^2 - r_2^2\right]}\right]$$

$$n = \frac{57296}{0.1 \times 0.35 \times \pi \times [70^2 - 40^2] \times 55}$$

$\boxed{n = 2.87}$ or say 3.

But n cannot be odd, it has to be even therefore,

$$\boxed{n = 4}$$

also, $n_1 + n_2 - 1 = n$

$$\boxed{n_1 + n_2 = 4 + 1 = 5}$$

n_1– bronze plates – 2 plates

n_2 – steel plates – 3 plates

2. Actual Axial force,

Actual torque per pair,

$$T_a = \frac{57296}{4}$$

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$$= 14324 \text{ N-mm}$$

$$T_a = \mu w_a \times Rm$$

$$W_a = \frac{14324}{0.1 \times 55}$$

$$= \boxed{2604.5 \text{ N}}$$

3. Actual Average Pressure

$$P_{av} = \frac{W_a}{\pi[r_1^2 - r_2^2]} = \frac{2604.5}{\pi[70^2 - 40^2]}$$

$$\boxed{0.25\, N/m}$$

4. Actual maximum pressure

$$P_{max} = \frac{C}{r_2}$$

We have to calculate cactula,

$$C_a = \frac{W_a}{2\pi(r_1 - r_2)}$$

$$= 13.818$$

$$P_{max} = \frac{13.818}{40}$$

$$= \boxed{0.345 \text{ N/mm}^2}$$

Example 3

Calculate the power transmitted if each contact surface wear by 1.25 mm and there are total 6 spring each of stiffness 13 KN/m assuming uniform wear.

$$n = 4$$

Total number of surface = 4 × 2 = 8.

Total wear = 8 × 1.25 = 10 mm = 0.01 m

Total steffness = $13 \times 10^3 \times 6$

Reduction in axial force due to wear,

$$= 13 \times 10^3 \times 6 \times 0.01 = 780 \text{ N}$$

New axial force, $= 2604.5 - 780$

$$= 1824.5 \text{ N}$$

$$T = n\mu WR_m$$

$$= 4 \times 0.1 \times 1824.5 \times (0.055)$$

$$= 40.139 \text{ N-m}$$

$$P = T \times w$$

$$= \frac{T \times 2\pi N}{60}$$

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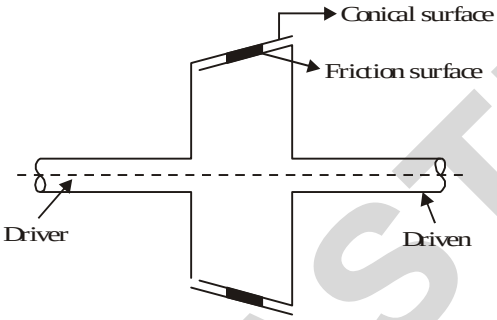
$$= \frac{40.139 \times 2 \times \pi \times 750}{60}$$

$$= 3152.51 \text{ W}$$

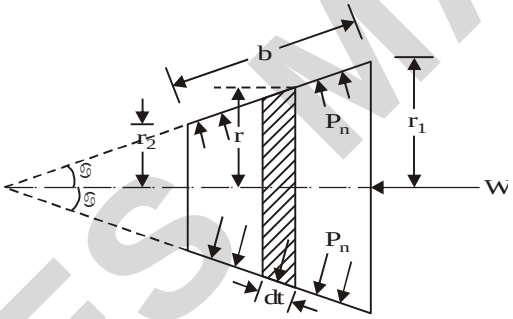
or $\boxed{3.152 \text{ kW}}$

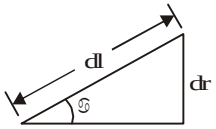
Cone Clutch

In cone clutch the driver is keyed to the driving shaft by sunk key. Driven is connected to driven shaft through feather key



Desgin





$$\sin\alpha = \frac{dr}{dl}$$

$$dl = \frac{dr}{\sin\alpha} = dr.\text{cosec}\,\alpha.$$

Torque, $T_r = 2\pi\mu r_m \text{cosec}\,\alpha.r^2 dr$

where,

$$\boxed{P_n = \frac{W}{\pi[r_1^2 - r_2^2]}}$$

1. Uniform pressure case

$$\boxed{T = n\mu w \,\text{cosec}\,\alpha\, R_m}$$

where, $R_m = \frac{2}{3}\left[\frac{r_1^3 - r_2^2}{r_1^2 - r_2^2}\right]$

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2. Uniform wear case

$$\boxed{P_r.r = C}$$

$$\boxed{C = \frac{W}{2\pi[r_1 - r_2]}}$$

$$\boxed{T = n\mu w \cos\alpha R_m}$$

where, $R_m = \frac{r_1 + r_2}{2}$

Face width is b,

$$\boxed{r_1 - r_2 = b\sin\alpha}$$

also, $P_n = \frac{W}{\pi[r_1^2 - r_2^2]}$

$$= \frac{W}{\pi(r_1 + r_2)(r_1 - r_2)} = \frac{W}{\pi \times 2R_m \times b\sin\alpha}$$

$$\boxed{P_n = \frac{W}{2\pi R_m b\sin\alpha}} \quad \text{....(1)}$$

Also torque is,

$T = \mu \times W \operatorname{cosec}\alpha \times R_m$ – from equation (1)

$= \mu \times r_m \times 2\pi R_m\, b\sin\alpha \times R_m \times \operatorname{cosec}\phi\alpha$

$$\boxed{T = 2\pi\mu\rho_n R^2 b}$$

Axial force required to engage the clutch is,

$$\boxed{W_e = w\operatorname{cosec}\alpha[\sin\alpha + \mu\cos\alpha]}$$

Axial force required to disengage the clutch,

$$\boxed{W_d = w\operatorname{cosec}\alpha(\mu\cos\alpha - \sin\alpha)}$$

Example 4

Mean diameter of friction sruface 80 mm and semi angle of clutch = 15°

μ = 0.3

W = 200 N

N = 900 r.p.m

The clutch is employed to connect an electric motor and a feywhell. The mass of flywheel is 145 kg and radius of gyration is 160 mm. Calculate time reqired for the flywheel to attain full speed and also energy lost in slipping. Also find the torque required to produce slipping of clutch.

Sol. $w = \frac{2\pi N}{60}$

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$$= \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

$w_f = w_0 + \alpha t$

$T = I \times \alpha.$ or

$$\alpha = \frac{T}{I}$$

Torque, $T = \mu \times w\text{cosec}\alpha \times R_m$

$= 0.3 \times 200 \times \text{cosec } 15° \times 40$

$= 9.273$ N-m

$I =$ Moment of Inertia

$= mK^2 = 14 \times (0.16)^2$

$= 0.3584$ Kg.

$\boxed{\alpha = 25.87 \text{ rad/s}^2}$

$w_f = w_i + \alpha t$

$94.26 = 0 + 25.87 \times t$

$\boxed{t = 3.64 \text{ s}}$

(b) Energy lost in shipping of clutch

= Torque × angular movement = T × θ

θ = Average w × time

$$= \left(\frac{0.94.26}{2}\right) \times 3.64$$

= 171.6 rad

$\boxed{E = T \times \theta = 9.273 \times 171.6 = 1591 \text{ N-m}}$

Centrifugal Clutch

Driver and driven are engaged with the help of centrifugal force generated by rotation of shoes outer surface of which is covered with friction material. These shoes are backed by springs. When the centrifugal force is more than the spring force the driven and driven are engaged.

Design

$F_C =$ Centrifugal force

$= mw^2r$

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