AFTERNOON

Solution that should be challenged by students for GATE 2024 CIVIL

31. Question ID (6420084811) [MSQ, 2 Marks]

Three vectors $\vec{p}, \vec{q}$ and $\vec{r}$ are given as

$\vec{p} = \hat{i} + \hat{j} + \hat{k}$ $\quad\quad$ $\vec{q} = \hat{i} + 2\hat{j} + 3\hat{k}$ $\quad\quad$ $\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Which of the following is/are CORRECT?

(a) $\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \bullet \vec{r})\vec{q} - (\vec{p} \bullet \vec{q})\vec{r}$

(b) $\vec{r} \bullet (\vec{p} \times \vec{q}) = (\vec{q} \times \vec{p}) \bullet \vec{r}$

(c) $\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \times \vec{q}) \times \vec{r}$

(d) $\vec{p} \times (\vec{q} \times \vec{r}) + \vec{q} \times (\vec{r} \times \vec{p}) + \vec{r} \times (\vec{p} \times \vec{q}) = \vec{0}$

Sol. (a,b,d)

Option (a): $\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \cdot \vec{r})\,\vec{q} - (\vec{p} \cdot \vec{q})\,\vec{r}$

By property of vector tripple product above expansion is always true.

Option (b): $\vec{r} \cdot (\vec{p} \times \vec{q}) = (\vec{p} \times \vec{p}) \cdot \vec{r}$

As we-know that

$$(\vec{p} \times \vec{q}) = -(\vec{q} \times \vec{p})$$

So, $\quad \vec{r} \cdot (\vec{p} \times \vec{q}) \neq (\vec{q} \times \vec{p}) \cdot \vec{r}$

But three is an exceptional case when

$$\vec{r} \cdot (\vec{p} \times \vec{q}) = (\vec{q} \times \vec{p})\,\vec{r} = 0$$

then $\quad \vec{r} \cdot (\vec{p} \times \vec{q}) = (\vec{q} \times \vec{p}) \cdot \vec{r}$

Now, according to given vectors

$$\vec{r} \cdot (\vec{p} \times \vec{q}) = (\vec{q} \times \vec{p}) \cdot \vec{r} = 0$$

So, in this $\vec{r} \cdot (\vec{p} \times \vec{q}) = (\vec{q} \times \vec{p}) \cdot \vec{r}$ is true

Option (c): $\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \times \vec{q}) \times \vec{r}$

$$\vec{p} \times (\vec{q} \times \vec{r}) \perp \vec{p}$$

$$(\vec{p} \times \vec{q}) \times \vec{r} \perp \vec{r}$$

So, both can not be equal option (c) false.

Option (d): $\vec{p} \times (\vec{q} \times \vec{r}) + \vec{q} \times (\vec{r} \times \vec{p}) + \vec{r} \times (\vec{p} \times \vec{q}) = 0$

$$\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \cdot \vec{r})\,\vec{q} - (\vec{p} \cdot \vec{q}) \cdot \vec{r} \quad \ldots(i)$$

$$\vec{q} \times (\vec{r} \times \vec{p}) = (\vec{q} \cdot \vec{p})\,\vec{r} - (\vec{q} \cdot \vec{r}) \cdot \vec{p} \quad \ldots(ii)$$

$$\vec{r} \times (\vec{p} \times \vec{q}) = (\vec{r} \cdot \vec{q}) \cdot \vec{p} - (\vec{r} \cdot \vec{p}) \cdot \vec{q} \quad \ldots(iii)$$

Adding (i), (ii) and (iii)

$$\vec{p} \times (\vec{q} \times \vec{r}) + \vec{q} \times (\vec{r} \times \vec{p}) + \vec{r} \times (\vec{p} \times \vec{q}) = 0$$

$$(\vec{p} \cdot \vec{q} = \vec{q} \cdot \vec{p}, \vec{q} \cdot \vec{r} = \vec{r} \cdot \vec{q}, \vec{r} \cdot \vec{p} = \vec{p} \cdot \vec{r})$$

This option is true.

Option (a, b, d) are correct.