IES MASTER
Institute for Engineers (IES/GATE/PSUs)

GATE 2024

CIVIL ENGINEERING

Detailed Solution

EXAM DATE: 04-02-2024
AFTERNOON SESSION (02:30 PM-05:30 PM)

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The colour of the Questions show the difficulty level of questions as per below mentioned colour code:

- Easy
- Moderate
- Hard

SECTION: GENERAL APTITUDE

1. If ' $\rightarrow$ ' denotes increasing order of intensity, then the meaning of the words [drizzle $\rightarrow$ rain $\rightarrow$ downpour] is analogous to [__ $\rightarrow$ quarrel $\rightarrow$ feud].

Which one of the given options is appropriate to fill the blank?

(a) dither (b) dodge

(c) bog (d) bicker

Sol. (d)

- Drizzle means intermittant rain
- Downpour means heavy rain
- Quarrel means an angry argument
- Feud means an angry and bitter argument
- Bicker means to argue about things that are not important.

Now its very clear that

drizzle $\rightarrow$ rain $\rightarrow$ downpour is analogous to

bicker $\rightarrow$ quarrel $\rightarrow$ feud

2. Five cubes of identical size and another smaller cube are assembled as shown in Figure A. If viewed from direction X, the planar image of the assembly appears as Figure B.

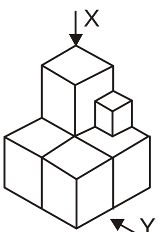


Figure A

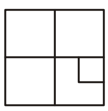
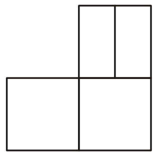
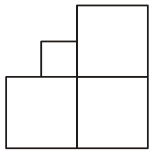
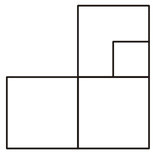


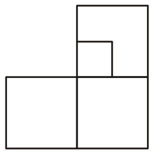
Figure B

If viewed from direction Y, the planar image of the assembly (Figure A) will appear as

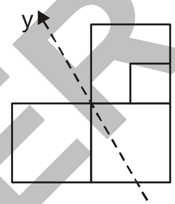
(a) 

(b) 

(c) 

(d) 

Sol. (c)



3. In the given text, the blanks are numbered (i)-(iv). Select the best match for all the blanks.

Yoko Roi stands ___(i)____ as an author for standing ___(ii)____ as an honorary fellow, after she stood ___(iii)____ her writings that stand ___(iv)____ the freedom of speech.

(a) (i) down (ii) out (iii) by (iv) in

(b) (i) down (ii) out (iii) for (iv) in

(c) (i) out (ii) down (iii) by (iv) for

(d) (i) out (ii) down (iii) in (iv) for

Sol. (c)

- stands out
- standing down
- by her writtings
- for the freedom of speech is the best filling

so option (c) is correct

4. A student was supposed to multiply a positive real number p with another positive real number q. Instead, the student divided p by q. If the percentage error in the student's answer is 80%, the value of q is

(a) 5 (b) $\sqrt{5}$

(c) $\sqrt{2}$ (d) 2

Sol. (b)

$$\text{Actual result} = pq$$

$$\text{wrong result} = p/q$$

$$\frac{\text{Wrong result}}{\text{Actual result}} = 0.2 \text{ (result reduced by 80\%)}$$

$$\frac{p/q}{pq} = 0.2 = \frac{2}{10} = \frac{1}{5}$$

$$\frac{1}{q^2} = \frac{1}{5} \Rightarrow q = \sqrt{5}$$

5. Statements:

1. All heroes are winners.
2. All winners are lucky people.

Inferences:

I. All lucky people are heroes.

II. Some lucky people are heroes.

III. Some winners are heroes.

Which of the above inferences can be logically deduced from statements 1 and 2?

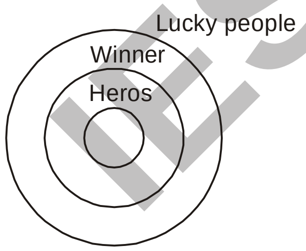
(a) Only III

(b) Only I and III

(c) Only II and III

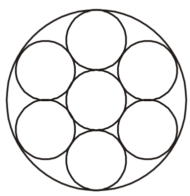
(d) Only I and II

Sol. (c)



From above Venn diagram its clear that (ii) and (iii) are valid deductions.

6. Seven identical cylindrical chalk-sticks are fitted tightly in a cylindrical container. The figure below shows the arrangement of the chalk-sticks inside the cylinder.



The length of the container is equal to the length of the chalk-sticks. The ratio of the occupied space to the empty space of the container is

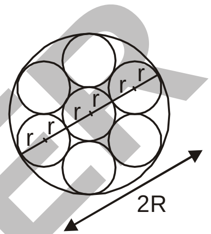
(a) 5/2

(b) 7/2

(c) 9/2

(d) 3

Sol. (b)



Let the radius of cylinder = R

and the radius of chalk = r

2r + 2r + 2r = 2R

r = (R/3)

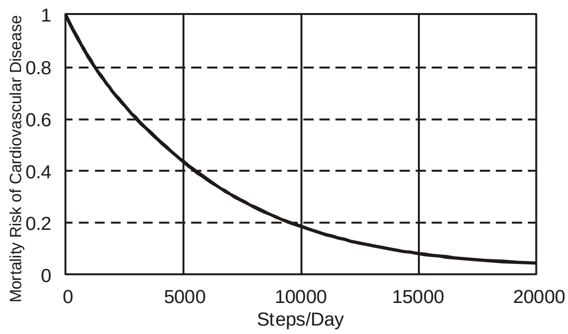
$$\text{Volume of cylinder} = \pi R^2 h$$

$$\text{Volume of chalk} = \pi\left(\frac{R}{3}\right)^2 h$$

$$\frac{\text{Volume of occupied space}}{\text{Volume of empty space}} = \frac{7\pi\frac{R^2}{9}h}{\pi R^2 h - \frac{7\pi R^2}{9}}$$

$$= \frac{7/9}{2/9} = \left(\frac{7}{2}\right)$$

7. The plot below shows the relationship between the mortality risk of cardiovascular disease and the number of steps a person walks per day. Based on the data, which one of the following options is true?



(a) The risk reduction on increasing the steps/day from 0 to 5000 is less than the risk reduction on increasing the steps/day from 15000 to 20000.

(b) For any 5000 increment in steps/day the largest risk reduction occurs on going from 0 to 5000.

(c) The risk reduction on increasing the steps/day from 0 to 10000 is less than the risk reduction on increasing the steps/day from 10000 to 20000.

(d) For any 5000 increment in steps/day the largest risk reduction occurs on going from 15000 to 20000.

Sol. (b)

Risk reduction value (0 – 5000)

= 1 – 0.45

= 0.55

which is maximum.

8. Visualize a cube that is held with one of the four body diagonals aligned to the vertical axis. Rotate the cube about this axis such that its view remains unchanged. The magnitude of the minimum angle of rotation is

(a) 60°

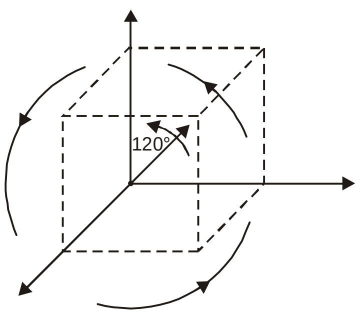
(b) 180°

(c) 120°

(d) 90°

Sol. (c)

When the cube is rotated that is held with one of the four body diagonals aligned to the vertical axis



9. If the sum of the first 20 consecutive positive odd numbers is divided by 20^2, the result is

(a) 1/2

(b) 20

(c) 2

(d) 1

Sol. (d)

$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 4^2$$
$$(1 + 3 + 5 + 4 \ldots\ 2 \text{ times}) = 20^2$$
$$\frac{20^2}{20^2} = 1$$

10. The ratio of the number of girls to boys in class VIII is the same as the ratio of the number of boys to girls in class IX. The total number of students (boys and girls) in classes VIII and IX is 450 and 360, respectively. If the number of girls in classes VIII and IX is the same, then the number of girls in each class is

(a) 150

(b) 175

(c) 250

(d) 200

Sol. (d)

Let no. of girls in 8^{th} = no. of girls in 9^{th}

= x

$$\frac{\text{No. of girls in } 8^{th} \text{ class}}{\text{No. of girls in } 9^{th} \text{ class}} = \frac{\text{No. of boys in } 9^{th} \text{ class}}{\text{No. of girls in } 9^{th} \text{ class}}$$

$$\frac{x}{450 - x} = \frac{360 - x}{x}$$
$$x^2 = (450 - x)(360 - x)$$
$$x^2 = 16200 - 450x - 360x + x^2$$
$$x = \left(\frac{16200}{810}\right)$$
$$= 200$$

SECTION: CIVIL ENGINEERING

1. Which one of the following products is NOT obtained in anaerobic decomposition of glucose?

(a) H_2S

(b) CO_2

(c) H_2O

(d) CH_4

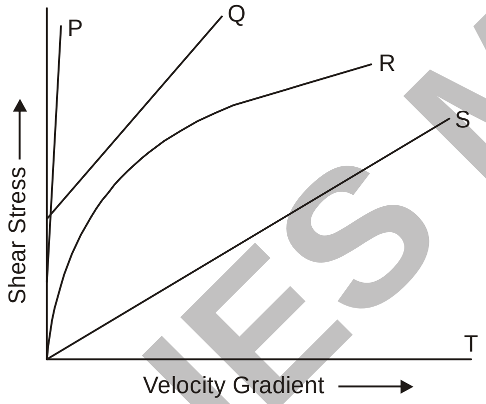
Sol. (a)

As glucose ($C_6H_{12}O_6$) does not contains sulphur hence anaerobic decomposition of glucose does not release H_2S.

Anaerobic decomposition of glucose gives CO_2, CH_4, H_2O as its byproducts.

Hence, (a) is the correct option

2. The following figure shows a plot between shear stress and velocity gradient for materials/fluids P, Q, R, S and T.



Which one of the following options is CORRECT?

(a) P $\rightarrow$ Real solid;Q $\rightarrow$ Ideal Bingham plastic

S $\rightarrow$ Newtonian fluid;T $\rightarrow$ Ideal Fluid

(b) P $\rightarrow$ Real solid;Q $\rightarrow$ Newtonian Fluid

R $\rightarrow$ Ideal Bingham Plastic;T $\rightarrow$ Ideal Fluid

(c) P $\rightarrow$ Ideal Fluid;Q $\rightarrow$ Ideal Bingham Plastic

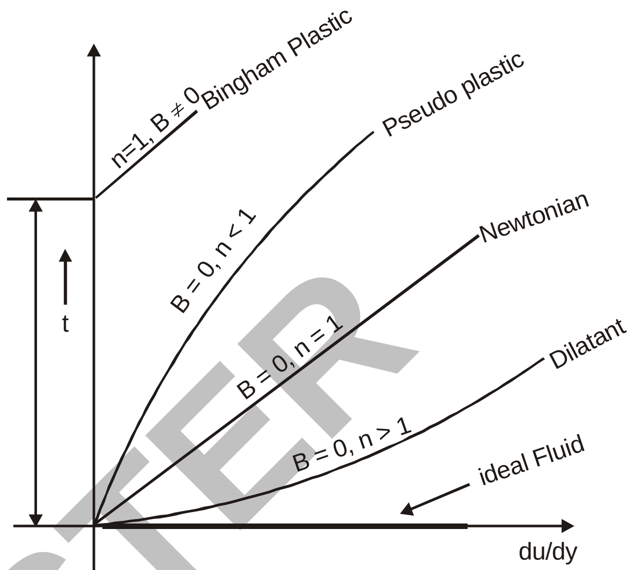
R $\rightarrow$ Non-Newtonian Fluid;S $\rightarrow$ Newtonian Fluid

(d) P $\rightarrow$ Ideal Fluid;Q $\rightarrow$ Ideal Bingham Plastic

R $\rightarrow$ Non-Newtonian Fluid;T $\rightarrow$ Real solid

Sol. (a)

B = Initial yield stress (i.e., B $\neq$ 0)



- Various types of newtonian & non-newtonian fluids are shown in the figure.
- Fluids which obeys Newton's law of viscosity $\left(\tau = \mu \frac{du}{dy}\right)$ are called **Newtonian Fluids** and those fluids which do not obey this rule are called **Non-Newtonian Fluids**.
- General relationship between shear stress and velocity gradient is given by

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$

- In the figures shown above, slope of the curve is called apparent viscosity.
- Fluid for which apparent viscosity increases with du/dy are called **Dilatant.**
- Dilatant fluids are also called shear thickening fluids. Examples of dilatant fluids are solution with suspended starch or sand, sugar in water.
- Fluids for which apparent viscosity decreases with du/dy are called **Pseudo Plastic**.
- Pseudo plastic fluid are also called shear thinning fluid. Examples are paints, polymer solutions, blood, paper pulp, syrup, molasses, milk, gelatine.

- Bingham Plastic (ideal plastic) fluids require a certain minimum shear stress t_y (yield stress) before they start flowing. Examples : tooth paste, sewage sludge, drilling mud have time dependent Newtonian Behaviour.

3. The second derivative of a function F is computed using the fourth-order Central Divided Difference method with a step length h.

The CORRECT expression for the second derivative is

(a) $\frac{1}{12h^2}\left[-f_{i+2}+16f_{i+1}-30f_i+16f_{i-1}-f_{i-2}\right]$

(b) $\frac{1}{12h^2}\left[-f_{i+2}+16f_{i+1}-30f_i+16f_{i-1}-f_{i-2}\right]$

(c) $\frac{1}{12h^2}\left[-f_{i+2}-16f_{i+1}+30f_i-16f_{i-1}-f_{i-2}\right]$

(d) $\frac{1}{12h^2}\left[f_{i+2}+16f_{i+1}-30f_i+16f_{i-1}-f_{i-2}\right]$

Sol. (a)

The second derivative of a function of using fourth-order central divided difference method is given by

$$f''(x) = \frac{\begin{array}{c} f(x+2h)+16f(x+h)-30f(x) \\ +16f(x-h)-f(x-2h) \end{array}}{12h}$$

where, h – 1 step size.

4. The longitudinal sections of a runway have gradients as shown in the table.

End of end for sections or runway(m)	Graident(%)
0 to 200	+1.0
200 to 600	–1.0
600 to 1200	+0.8
1200 to 1600	+0.2
1600 to 2000	–0.5

Consider the reduced level (RL) at the starting point of the runway as 100m. The effective gradient of the runway is

(a) 0.18%

(b) 0.02%

(c) 0.35%

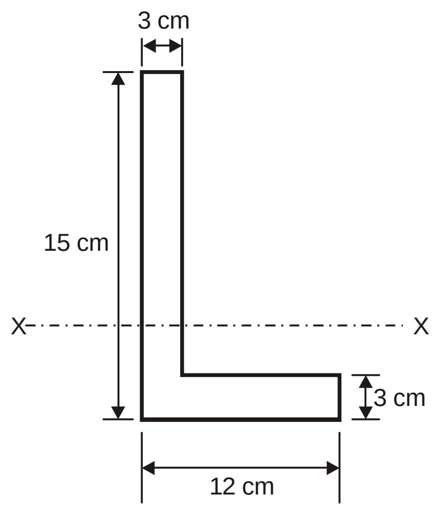
(d) 0.28%

Sol. (d)

End to end section of runway(m)	Graident (%)	RL (m)
0		100(given)
200	+1	$100+200\times\frac{1}{100}=102$
600	–1	$102-400\times\frac{1}{100}=98$
1200	+0.8	$98+600\times\frac{0.8}{100}=102.8$
1600	+0.2	$102.8+400\times\frac{0.2}{100}=103.6$
2000	–0.5	$103.6-400\times\frac{0.5}{100}=101.6$

$$\text{Effective gradient} = \frac{103.6-98}{2000}\times 100 = 0.28\%$$

5. The steel angle section shwon in the figure has elastic section modulus of 150.92 cm^3 about the horizontal X-X axis, which passes through the centroid of the section.

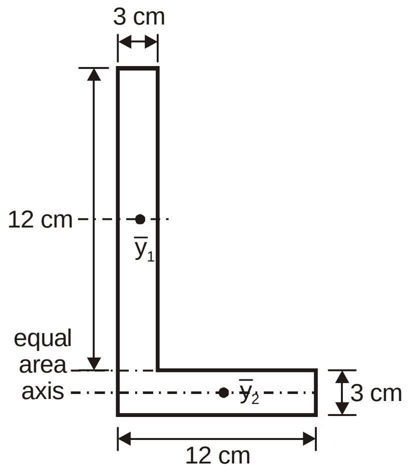


(Figure NOT to scale)

The shape factor of the section is ________ (rounded off to 2 decimal places)

Sol. (1.79)

$$\text{Shape factor} = \frac{Z_p}{Z_e}$$



$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$= \frac{2 \times 12 \times 3}{2}(6 + 1.5) = 270$$

$$SF = \frac{270}{150.92} = 1.79$$

6. To finalize the direction of a survey, four surveyors set up a theodolite at a station P and performed all the temporary adjustments. From the station P, each of the surveyors observed the bearing to a tower located at station Q with the same instrument without shifting it. The bearing observed by the surveyors are 30°30'00", 30°29'40", 30°30'20" and 30°31'20". Assuming that each measurement is taken with equal precision, the most probable value of the bearing is

(a) 30°31'20"

(b) 30°29'40"

(c) 30°30'20"

(d) 30°30'00"

Sol. (c)

Most probable value with equal weightage

$$= 30^\circ\left[\frac{30' + 29'40'' + 30'20'' + 31'20''}{4}\right] = 30^\circ 30' 20''$$

7. Consider two ordinary differential equations (ODEs):

$$P : \frac{dy}{dx} = \frac{x^4 + 3x^2y^2 + 2y^4}{x^3y}$$

$$Q : \frac{dy}{dx} = \frac{-y^2}{x^2}$$

Which one of the following options is CORRECT?

(a) P is homogeneous ODE and Q is an exact ODE.

(b) P is a nonhomogenous ODE and Q is not an exact ODE

(c) P is homogeneous ODE and Q is not an exact ODE

(d) P is a non homogeneous ODE and Q is an exact ODE

Sol. (b)

$$P : \frac{dy}{dx} = \frac{x^4 + 3x^2y^2 + 2y^4}{x^3y}$$

$$\frac{dy}{dx} = \left(\frac{x^4}{x^3y}\right) + \left(\frac{3x^2y^2}{x^3y}\right) + \left(\frac{2y^4}{x^3y}\right)$$

$$\frac{dy}{dx} = \left(\frac{x}{y}\right) + 3\left(\frac{y}{x}\right) + 2\left(\frac{y}{x}\right)^3$$

$$\frac{dy}{dx} = \left(\frac{1}{y/x}\right) + 3\left(\frac{y}{x}\right) + 2\left(\frac{y}{x}\right)^3$$

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

So P is a homogenous ODE

$$Q : \frac{dy}{dx} = \frac{-y^2}{x^2}$$

$x^2dy = -y^2dx$

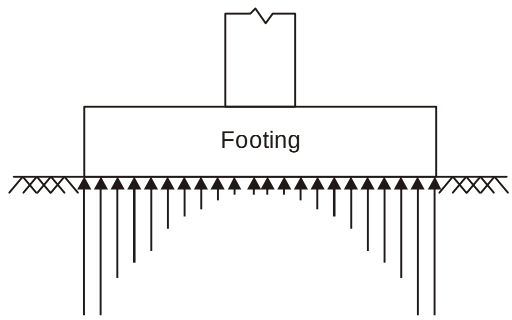
$y^2dx + x^2dy = 0$

$M = y^2 \quad \frac{dM}{\partial y} = 2y$

$N = x^2 \quad \frac{\partial N}{\partial x} = 2x$

So Q is non exact ODE

8. The contact presure distribution shown in the figure belongs to a



(a) flexible footing resting on a cohesive soil

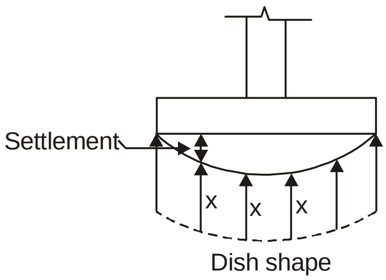
(b) rigid footing resting on a cohesive soil

(c) rigid footing resting on a cohesionless soil

(d) flexible footing resting on a cohesionless soil

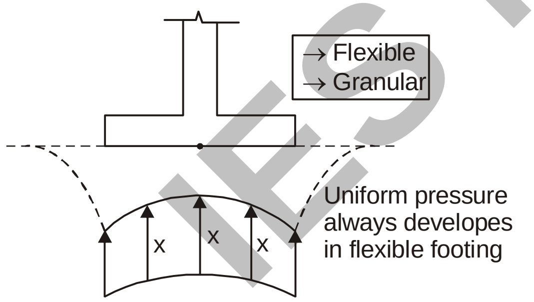
Sol. **(b)**

1. **Flexible footing over clayey soil:** In flexible footing, the contact pressure at the interface between footing and soil is uniformly distributed producing dish-shape pattern in clayey soil.

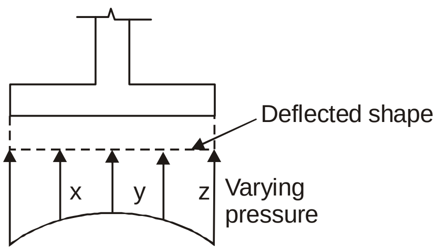


Flexible footing over clayey soil

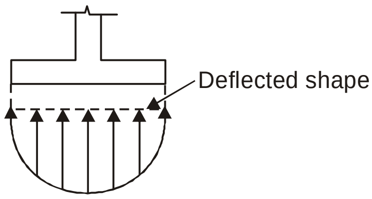
2. **Flexible footing over Granular soil:** In granular soil, modulus of elasticity (E_S) varies across the width being maximum at the centre and minimum at edge. As E is maximum at centre, defflection is less at centre. As E is less at edge deflection is more at edge.



3. **Rigid footing on Clayey soil:** In case of flexible footing, deflection is more at centre. Hence pressure developed at centre is less. Deflection is less in flexible footing at edge, hence in rigid footing pressure developed is more at edge.



4. **Rigid footing on Granular soil**



9. Various stresses in jointed plain concrete pavement with slab size of 3.5m × 4.5m are denoted as follows:

Wheel load stress at interior = S_{wl}^{i}

Wheel load stress at edge = S_{wl}^{e}

Wheel load stress at corner = S_{wl}^{c}

Warping stress at interior = S_{t}^{i}

Warping stress at edge = S_{t}^{e}

Warping stress at corner = S_{t}^{c}

Frictional stress between slab and supporting layer = S_f

The critical stress combination in the concrete slab during a summer midnight is

(a) $S_{wl}^{c} + S_{t}^{c}$

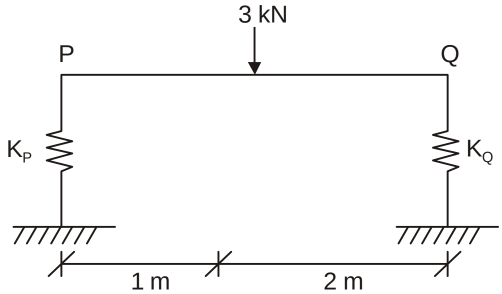
(b) $S_{wl}^{c} + S_{t}^{c} + S_f$

(c) $S_{wl}^{e} + S_{t}^{e} + S_f$

(d) $S_{wl}^{e} + S_{t}^{e} + S_f$

Sol. **(a)**

10. A 3 m long, horizontal, rigid, uniform beam PQ has negligible mass. The beam is subjected to a 3kN concentrated vertically downward force at 1m from P, as shown in the figure. The beam is resting on vertical linear springs at the ends P and Q. For the spring at the end P, the spring constant K_p = 100kN/m.



(Figure NOT to scale)

If the beam does not rotate under the application of the force and displaces only vertically, the value of the spring constant K_Q (in kN/m) for the spring at the end Q is

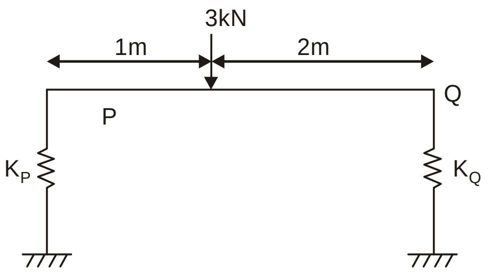
(a) 150

(b) 50

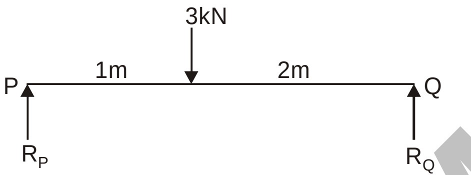
(c) 100

(d) 200

Sol. (b)



FBD



$\Sigma M_Q = 0 \Rightarrow R_P \times 3 - 3 \times 2 = 0$

$R_P = 2$ kN & $R_Q = 1$ kN

As beam PQ is rigid & for no rotation settlement at both P & Q should be same.

$$\Delta_P = \Delta_Q$$

$$\frac{R_P}{K_P} = \frac{R_Q}{K_Q}$$

$$\Rightarrow K_Q = \frac{R_Q}{R_P} \times K_P = \frac{1}{2} \times 100 = 50 \text{kN/m}$$

11. The function $f(x) = x^3 - 27x + 4, \; 1 \le x \le 6$ has

(a) Inflection point

(b) Saddle point

(c) Minima point

(d) Maxima point

Sol. (c)

$$f(x) = x^3 - 27x + 4$$

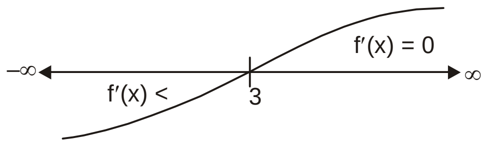
$$f'(x) = 3x^2 - 27$$

$$f'(x) = 0$$

$$3x^2 - 27 = 0$$

$$x = \pm 3$$

$$x = 3 \in (1, 6)$$



So at x = 3 function has point at local minima.

12. The structural design method that DOES NOT take into account the safety factors on the design load is

(a) working stress method

(b) ultimate load method

(c) load factor method

(d) limit state method

Sol. (a)

In working stress method, safety is accounted for by considering factor of safety in material strength only and no factor is considered in load.

Hence, the correction option (a).

13. A partial differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

is defined for the two-dimensional field T : T (x,y), inside a planar square domain of size 2m × 2m. Three boundary edges of the square domain are maintained at value T = 50, whereas the fourth boundary edge is maintained at T = 100.

The value of T at the center of the domain is

(a) 75.0

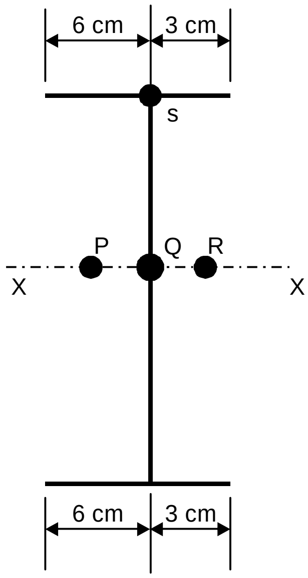
(b) 50.0

(c) 87.5

(d) 62.5

Sol. (d)

14. For a thin-walled section shown in the figure, points P, Q and R are located on the major bending axis X-X of the section. Point Q is located on the web whereas point S is located at the intersection of the web and the top flange of the section.



(Figure NOT to scale)

Qualitatively, the shear center of the section lies at

(a) Q

(b) P

(c) S

(d) R

Sol. (d)

15. A reinforced concrete pile of 10m length and 0.7m diameter is embedded in a saturated pure clay with unit cohesion of 50 kPa. If the adhesion factor is 0.5, the net ultimate uplift pullout capacity (in kN) of the pile is ________. (rounded off to the nearest integer).

Sol. (550 kN)

Given,

Length of pile (L) = 10m

Diameter of pile (D) = 0.7m

Cohesion of clay (C_4) = 50 kPa

Adhesion factor (α) = 0.5

Pullout capacity (P) = ?

Uplift pull out capacity = skin friction resistance + wt. of pile

For clay, (P) skin friction resistance = $f_s A_s$

$$P = \alpha C_u A_s + W$$

Taking unit wt of concrete = 25 kN/m^3

$$P = 0.5 \times 50 \times \pi \times 0.7 \times 10 + \frac{\pi(0.7)^2}{4} \times \gamma_c$$

$$P = 0.5 \times 50 \times \pi \times 0.7 \times 10 + \frac{\pi(0.7)^2}{4} \times 25$$

$$= 550 \text{ kN}$$

16. In general, the outer edge is raised above the inner edge in horizontal curves for

(a) Highways and Railways only

(b) Highways only

(c) Railways and Taxiways only

(d) Highways, Railways and Taxiways

Sol. (a)

17. What is the CORRECT match between the air pollutants and treatment techniques given in the table?

Air pollutants	Treatment techniques
P. NO_2	i. Flaring
Q. SO_2	ii. Cyclonic separator
R. CO	iii. Lime scrubbing
S. Particles	iv. NH_3 injection

(a) P-iv, Q-iii, R-i, S-ii

(b) P-i, Q-ii, R-iii, S-iv

(c) P-ii, Q-i, R-iv, S-iii

(d) P-ii, Q-iii, R-iv, S-i

Sol. (a)

(I) Following treatment techniques are used to remove particulate Matter (PM).

(a) Settling chambers

(b) Inertial or Impact separators

(c) Centrifugal separators or Cyclonic separators

(d) Filters

(e) Electrostatic precipitators

(f) Scrubbers or wet collectors

(II) Following treatment techniques are used to remove/control gaseous contaminants

(a) Combustion techniques

- This method is used when gases are of organic nature.
- Equipments used in combustion are:

(i) vapour incinerators

(ii) after burners

(iii) Flares (process is called as flaring)

Note: Flaring (i.e. combustion method) is suitable for the removal of carbon monoxide because during combustion, carbon reacts with carbon monoxide to form CO_2.

(b) Absorption

- In this method, effluent gases are passed through absorbers (or scrubbers) which contain liquid absorbents that remove various gaseous pollutants.

Gaseous pollutant	Common absorbent used as solid form
SO_2	Dimethylaniline, ammonium sulphite, sodium sulphite etc.
H_2S	Mixture of NaOH & phenol, soda Ash etc.
NO_x	Water, aqueous nitric acid
HF	Water, NaOH

(c) Adsorption technique

- In this method, the effluent gases are passed through adsorbers which contain solids of porous structure.

Gaseous pollutant	Common absorbent used as solid form
SO_2	Pulverised limestone or Dolomite
H_2S	Iron oxide
NO_x	Silica gel
HF	Lump limestone

Hence, correct option is (a)

18. The statements P and Q are related to matrices A and B, which are conformable for both addition and multiplication.

$$P : (A+B)^T = A^T + B^T$$

$$Q : (AB)^T = A^T B^T$$

Which one of the following options is CORRECT?

(a) Both P and Q are false

(b) Both P and Q are true

(c) P is false and Q is true

(d) P is true and Q is false

Sol. (d)

According to properties of a matrix

(i) $(A + B)^T = A^T + B^T$

The sum of transpose of matrices is equal to the transpose of the sum of two matrices.

(ii) $(AB)^T = B^T A^T$

The product of the transpose of two matrices in reverse order is equal to the transpose of the product of them.

Hence, option (d) is correct.

19. What is the CORRECT match between the survey instruments/parts of instruements shown in the table and the operations carried out with them?

Instruments / Parts of instruments	Operations
P. Bubble tube	i. Tacheometry
Q. Plumb bob	ii. Minor movements
R. Tangent screw	iii. Centering
S. Stadia cross-wire	iv. Levelling

(a) P-ii, Q-iii, R-iv, S-i

(b) P-iv, Q-iii, R-ii, S-i

(c) P-iii, Q-iv, R-i, S-ii

(d) P-i, Q-iii, R-ii, S-iv

Sol. (b)

- Bubble tube is used for levelling
- Plumb bob is used for centering

- Tangent screw in theodolite is used for minor movements
- Stadia cross wire is used in Tacheometry

20. Consider the statements P and Q.

P: In a Pure project organization, the project manager maintains complete authroity and has maximum control over the project.

Q: A matrix organization structure facilitates quick response to changes, conflicts, and project needs.

Which one of the following options is CORRECT?

(a) P is false and Q is true

(b) Both P and Q are false

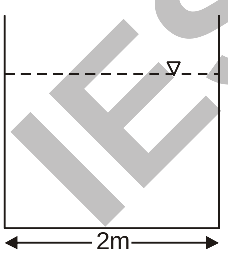
(c) Both P and Q are true

(d) P is true and Q is false

Sol. (d)

21. A 2m wide rectangular channel is carrying a discharge of 30 m^3/s at a bed slope of 1 in 300. Assuming the energy correction factor as 1.1 and acceleration due to gravity as 10 m/s^2, the critical depth of flow (in meters) is ________ (rounded off to 2 decimal places)

Sol. (2.91)



Discharge (Q) = $30 m^3/s$

Width of channel (B) = 2m

Bed slope = 1 in 300

Energy correction factor (α) = 1.1

g = 10 m/s^2

Discharge per unit width (q) = $\dfrac{Q}{B}$

$$= \frac{30}{2} = 15\, m^3/\text{sec}/m$$

Critical depth does not depend on slope or the roughness of channel.

$$\text{Critical depth } (y_c) = \left(\frac{q^2 \cdot \alpha}{g}\right)^{1/3}$$

$$= \left(\frac{1.1 \times 15^2}{10}\right)^{1/3} = 2.914m \simeq 2.91m$$

Note:

$$\text{Total energy (E)} = y + \frac{\alpha V^2}{2g}$$

$$= y + \frac{\alpha q^2}{y^2 \times 2g}$$

$$\frac{dE}{dy} = 1 + \frac{\alpha q^2}{2g}\frac{(-2)}{y^3}$$

For critical low, putting $\dfrac{dE}{dy} = 0$

$$\frac{\alpha q^2}{g y_c^3} = 1$$

$$\therefore \quad \boxed{y_c = \left(\frac{\alpha q^2}{g}\right)^{1/3}}$$

22. Which one of the following saturated fine-grained soils can attain a negative Skempton's pore pressure coefficient (A)?

(a) Quick clays

(b) Lightly-consolidated clays

(c) Normally-consolidated clays

(d) Over-consolidated clays

Sol. (d)

- Value of skempton's pore pressure coefficient (A) may be as large as 2 to 3 for very loose saturated fine sand.

  It can be (–0.5 to 0) for heavily over consolidated clays.

  For normally consolidated clays A = 0.5 – 1

- For OC clays, A = f (OCR), for heavily over consolidated clays, A < 0.

23. For a reconnaisssance survey, it is necessary to obtain vertical aerial photographs of a terrain at an average scale of 1:13000 using a camera. If the permissible flying height is assumed as 3000m above a datum and the average terrain elevation is 1050 m above the datrum, the required focal length (in mm) of the camera is

(a) 125

(b) 100

(c) 150

(d) 200

Sol. (c)

$$\text{Scale} = \frac{1}{13000}$$

Flying height (H) = 3000m

Elevation (h) = 1050m

$$S = \frac{f}{H-h}$$

$$\frac{1}{13000} = \frac{f}{3000-1050}$$

$$\therefore \quad f = 0.15\text{m} = 150 \text{ mm}$$

24. Consider teh following data for a project of 300 days duration.

Budgeted cost of work scheduled (BCWS) = Rs. 200

Budgeted cost of work performed (BCWP) = Rs. 150

Actual cost of work performed (ACWP) = Rs. 190

The 'schedule variance' for the project is

(a) (–)Rs. 50

(b) (+)Rs. 50

(c) (+)50 days

(d) (–)50 days

Sol. (a)

Schedule variance = BCWP – BCWS

= 150 – 200

= –50

25. A simply supported, uniformly loaded, two-way slab panel is torsionally unrestrained. The effective span lengths along the short span (x) and long span (y) directions of the panel are l_x and l_y respectively. The design moments for the reinforcements along the x and y directions are M_{ux} and M_{uy} respectively. By using Rankine-Grashoff method, the ratio M_{ux}/M_{uy} is proportional to

(a) l_y/l_x

(b) l_x/l_y

(c) $(l_y/l_x)^2$

(d) $(l_x/l_y)^2$

Sol. (c)

As per Rankine Grashoff method

$$M_{ux} = \frac{r^4}{8(1+r^4)} w\ell_x^2$$

$$M_{uy} = \frac{r^2}{8(1+r^4)} w\ell_x^2$$

Here, $$r = \frac{\ell_y}{\ell_x}$$

$$\Rightarrow \quad \frac{M_{ux}}{M_{uy}} = r^2 = \left(\frac{\ell_y}{\ell_x}\right)^2$$

Hence, the correct option is (c)

26. The expression for computing the effective interest rate (i_{eff}) using continous compounding for a nominal interest rate of 5% is

$$i_{eff} = \lim_{m\to\infty}\left(1+\frac{0.05}{m}\right)^m - 1$$

The effective interest rate (in percentage) is _______ (rounded off to 2 decimal places).

Sol. (5.127%)

$$i_{eff} = \lim_{m\to\infty}\left(1+\frac{0.05}{m}\right)^m - 1$$

$$= e^{\lim_{m\to\infty}\left(1+\frac{0.05}{m}\right)\times m - 1}$$

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AIR 24 CE PRASHANT TRIPATHI
AIR 25 CE SOURABH MISHRA
AIR 26 CE AMAN
AIR 28 CE POKHRAJ NAYAK
AIR 29 CE BHUPENDRA MEENA
AIR 31 CE MERUVA ANIL KUMAR REDDY
AIR 32 CE NILESH KUMAR SINGH
AIR 34 CE ARPIT NAUTIYAL

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AIR 45 CE SACHIN SINGHAL
AIR 46 CE AYUSH JAIN
AIR 47 CE ADIL SABIH

AIR 48 CE ABHISHEK KUMAR
AIR 50 CE PAWAN SINGH NEGI
AIR 51 CE SATYAM
AIR 52 CE SAHIL SIJARIYA
AIR 54 CE AATESH KUMAR MEENA
AIR 55 CE SUBHADRA SUBHADARSHINI
AIR 56 CE SHASHANK SHEKHAR MISHRA
AIR 57 CE BHAWANA JOSHI
AIR 58 CE SAKSHAM SHARMA
AIR 59 CE KAMIDI NITESH REDDY
AIR 61 CE LOHARKAR KARISHMA SUNIL

AIR 62 CE SANDEEP KUMAR
AIR 63 CE UTKARSH MISHRA
AIR 64 CE SHUBHAM PAWAR
AIR 66 CE SHIVASHISH GAUTAM
AIR 67 CE YASHWANT KUMAR
AIR 68 CE SHAHEZAMA
AIR 71 CE BOLLA USHNEESH NANDAN
AIR 72 CE SAUMYA CHANDRA
AIR 73 CE ANKIT TAILOR
AIR 74 CE HEMANT SINGH
AIR 75 CE GARIMA KATIYAR

AIR 76 CE REALTU KUMAR
AIR 78 CE VAIBHAV BIHANI
AIR 79 CE SATYAM DEWANGAN
AIR 81 CE ATIKANT KUMAR
AIR 82 CE MAYANK RANJAN
AIR 83 CE SATYAM KUMAR
AIR 85 CE SAURAV KUMAR
AIR 89 CE SATYA PRABHAT RANJAN
AIR 92 CE NAKKA DURGA PRASANNA KUMAR
AIR 93 CE RISHITA SINGH
AIR 96 CE SANDEEP SONI

AIR 97 CE ABHISHEK ANAND
AIR 98 CE PUSHPENDRA SRIVASTAVA
AIR 99 CE ANUPAMA
AIR 101 CE ROHIT KUMAR SINGH
AIR 103 CE RAHUL KUMAR MINA
AIR 104 CE RAJJAT SINGLA
AIR 105 CE RAKESH CHOUDHARY
AIR 107 CE RAVI VERMA
AIR 108 CE PREM KUMAR
AIR 109 CE RAVI KUMAR GUPTA
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$$= e^{\lim\limits_{m\to\infty}\left(\frac{0.05}{m}\times m\right)-1}$$

$$= e^{0.05} - 1 = 0.05127\%$$

27. In a sample of 100 heart partients, each patients has 80% chance of having a heart attack without medicine X. It clinically known that medicine X reduces the probability of having a heart attack by 50%. Medicine X is taken by 50 of these 100 patients. The probability that a randomly selected patient, out of the 100 patients, takes medicine X and has a heart attack is

(a) 40%

(b) 30%

(c) 20%

(d) 60%

Sol. (c)

Probability of having a heart attack without medicine = 80%

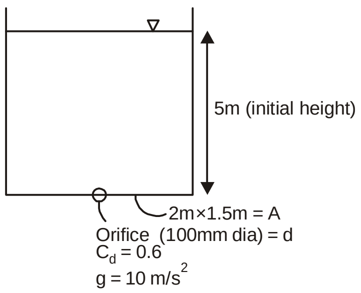
Probability of having a heart attack with medicine = (80%) × 0.5 = 40%

Total probability that a randomly selected patient out of 100 takes medicines X and has a heart attack.

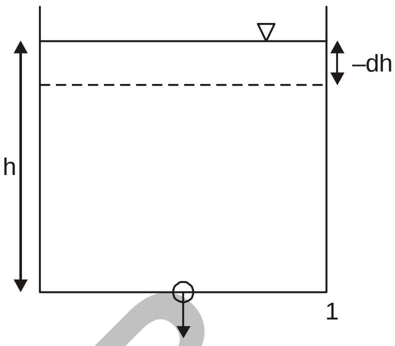
$$= \frac{50}{100}\times 0.4 = 0.2 = 20\%$$

28. A 2m × 1.5m tank of 6m height is provided with a 100mm diameter orifice at the center of its base. The orifice is plugged and the tank is filled up to 5m height. Consider the average value of discharge coefficient as 0.6 and acceleration due to gravity (g) as 10m/s^2. After unplugging the orifice, the time (in seconds) taken for the water level to drop from 5m to 3.5m under free discharge condition is ______. (rounded off to 2 decimal places).

Sol. (103.985 sec)



Let at any instant depth of liquid in tank is 'h' m and in time (dt), the depth falls by (–dh)



$$Q.dt = A_0(-dh)$$

$$\Rightarrow \left(C_d\cdot\left(\frac{\pi d^2}{4}\right)\sqrt{2gh}\right)dt = A_0(-dh)$$

$$t = \int dt = \frac{A_0}{C_d\cdot\frac{\pi d^2}{4}\sqrt{2g}}\int_{H_1}^{H_2}\left(\frac{-dH}{\sqrt{h}}\right)$$

$$t = \frac{A_0}{C_d\cdot\frac{\pi d^2}{4}\sqrt{2g}}\left[2\left(\sqrt{H_1}-\sqrt{H_2}\right)\right]$$

$$t = \left[\frac{2\times 1.5\text{m}^2}{0.6\times\frac{\pi}{4}(0.1)^2\,\text{m}^2\sqrt{2\times 10\frac{\text{m}}{\text{s}^2}}}\times 2\left(\sqrt{5}-\sqrt{3.5}\right)\text{m}^{1/2}\right]\text{sec}$$

$$\boxed{t = 103.985\,\text{sec}}$$

29. A round-bottom trianglular lined canal is to be liad at a slope of 1m in 1500, to carry a discharge of 25m^3/s. The side slopes of the canal cross-section are to be kept at 1.25H:1V. If Manning's roughtness coefficient is 0.013, the flow depth (in meters) will be in the range of

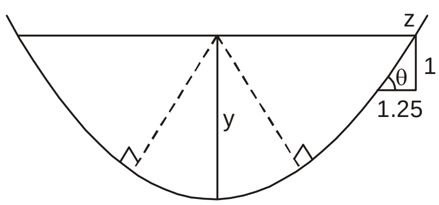
(a) 1.94 to 1.97

(b) 2.61 to 2.64

(c) 2.24 to 2.27

(d) 2.39 to 2.42

Sol. (d)



$$\cot\theta = 1.25$$

$$\theta = 0.675 \text{ rad}$$

$$n = 0.013$$

$$Q = 25 \text{ m}^3/\text{s},\ S = \frac{1}{1500}$$

Using manning's equation:

$$\text{Area} = y^2(\theta + \cot\theta)$$

$$A = 1.925\, y^2$$

$$R = \frac{y}{2}$$

$$Q = \frac{A}{n} R^{2/3} (S)^{1/2}$$

$$25 = \frac{1.925y^2}{0.013}\left(\frac{y}{2}\right)^{2/3}\left(\frac{1}{1500}\right)^{1/2}$$

$$\Rightarrow \quad y = 2.40 \text{ m}$$

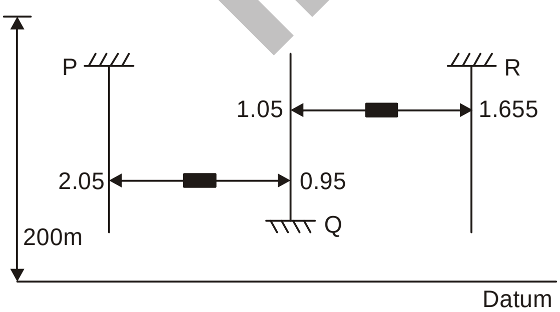
30. Differential levelling is carried out from point P (BM: +200.000m) to point R.

The reading taken are given in the table.

Points	Staff readings (m)		Remarks
	Back Sight	Fore Sight	
P	(–)2.050		BM: +200.000 m
Q	1.050	0.950	Q is a change point
R		(–)1.655	

Reduced level (in meters) of the point R is _____ (rounded off to 3 decimal places)

Sol. **(199.705)**



Using go through line (starting from P)

$$-2.05 + 0.1 + 1.655 = -0.295$$

(means R is 0.295 lower than P)

$\therefore$ RL of R = 200 – 0.295 = 199.705

31. Three vectors $\vec{p}, \vec{q}$ and $\vec{r}$ are given as

$$\vec{p} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{q} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Which of the following is/are CORRECT?

(a) $\vec{p}\times(\vec{q}\times\vec{r}) = (\vec{p}\bullet\vec{r})\vec{q} - (\vec{p}\bullet\vec{q})\vec{r}$

(b) $\vec{r}\bullet(\vec{p}\times\vec{q}) = (\vec{q}\times\vec{p})\bullet\vec{r}$

(c) $\vec{p}\times(\vec{q}\times\vec{r}) = (\vec{p}\times\vec{q})\times\vec{r}$

(d) $\vec{p}\times(\vec{q}\times\vec{r}) + \vec{q}\times(\vec{r}\times\vec{p}) + \vec{r}\times(\vec{p}\times\vec{q}) = \vec{0}$

Sol. **(a,b,d)**

(a) $\vec{p}\times(\vec{q}\times\vec{r}) = (\vec{p}\bullet\vec{r})\vec{q} - (\vec{p}\bullet\vec{q})\vec{r}$

(This is always true for any three given vectors)

(b) We know that $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ is always true but $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$ because $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

This can be true only when $\vec{a}\times\vec{b} = 0$

So, $\vec{r}.(\vec{p}\times\vec{q}) = \vec{r}.(\vec{q}\times\vec{p})$

$\vec{r}.(\vec{p}\times\vec{q}) = -\vec{r}.(\vec{p}\times\vec{q})$

This can be true if

$\vec{r}.(\vec{p}\times q) = 0$

$\vec{p}\times\vec{q} = \hat{i} - 2\hat{j} + j$

$(\vec{r}.\vec{p}\times\vec{q}) = 0$ this is true in this case

(c) $\vec{p}\times(\vec{q}\times\vec{r})$ can't be equal to $(\vec{p}\times\vec{q})\times\vec{r}$ because

$\vec{p}\times(\vec{q}\times\vec{r}) \perp \vec{p}$ and $(\vec{p}\times\vec{q})\times\vec{r}) \perp \vec{r}$

So, $(\vec{p}\times(\vec{q}\times\vec{r}) \neq (\vec{p}\times\vec{q}))\times\vec{r}$

(d) $\vec{p}\times(\vec{q}\times\vec{r}) + \vec{q}\times(\vec{r}\times\vec{p}) + \vec{r}\times(\vec{p}\times\vec{q}) = \vec{0}$

$(\vec{p}.\vec{r})\vec{q} - (\vec{p}.\vec{q})\vec{r} + (\vec{q}.\vec{p})\vec{p} - (\vec{q}.\vec{r})\vec{p} + (\vec{r}.\vec{q})\vec{p} - (\vec{r}.\vec{q})\vec{q}$

$0 = 0$

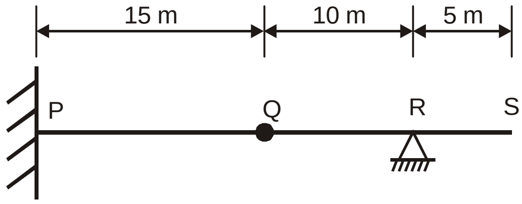
$\because \qquad \vec{p}.\vec{r} = \vec{r}.\vec{p}$

$\vec{p}.\vec{q} = \vec{q}.\vec{p}$

$\vec{q}.\vec{r} = \vec{r}.\vec{q}$

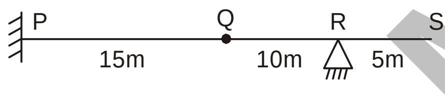
(Hence proved)

32. The horizontal beam PQRS shown in the figure has a fixed support at point P, an internal hinge at point Q, and a pin support at point R. A concentrated vertically downward load (V) of 10kN can act at any point over the entire length of the beam.



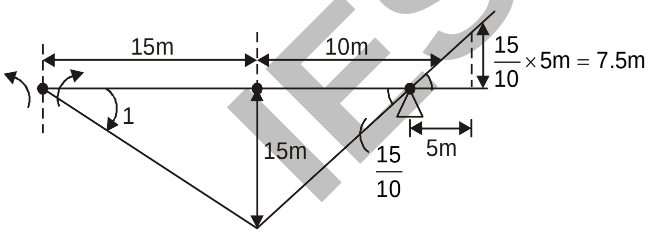
The maximum magnitude of the moment reaction (in kN.m) that can act at the support P due to V is ________ (in integer).

Sol. **(150 kNm)**



Beam is determinate

ILD for BM at 'P' is given by

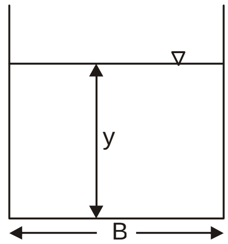


Hence when point load is at 'Q', the point of max ordinate for ILD for BM at P, BM at P will be max.

$\Rightarrow \quad \boxed{M_{P\max} = 10 \times 15 = 150\text{kN-m}}$

33. A rectangular channel is 4.0m wide and carries a discharge of 2.0 m³/s with a depth of 0.4m. The channel transitions to a maximum width contraction at a downstream location, without influencing the upstream flow condtions. The width (in meters) at the maximum contraction is ________ (rounded off to 2 decimal places)

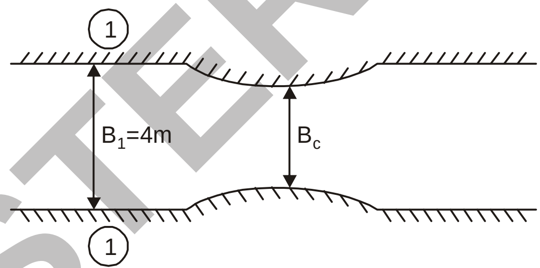
Sol. **(3.53)**



width of channel (B) = 4m

discharge (Q) = $2m^3/s$

depth of flow(y) = 0.4m



Specific energy at section (1) – (1) (E)

$$= y_1 + \frac{V_1^2}{2g}$$

$$= 0.4 + \frac{Q^2}{B_1^2 \times 2g \times y_1^2}$$

$$= 0.4 + \frac{2^2}{4^2 \times 2 \times 9.81 \times 0.4^2}$$

$$= 0.4796 \text{ m}$$

When channel section is contracted to minimum width and for constant discharge Q. The flow over contracted section will be critical flow.

$$E = E_c = \frac{3}{2} \cdot y_c$$

$$\therefore \quad 0.4796 = \frac{3}{2} \times y_c$$

$$\Rightarrow \quad y_c = 0.4796 \times \frac{2}{3} = 0.3197\text{m}$$

For critical flow condition, $\frac{Q^2 T}{gA^3} = 1$

$$\frac{Q^2 B_{min}}{g B_{min}^3 y_c^3} = 1$$

$$\Rightarrow \quad B_{min}^2 = \frac{Q^2}{9y_c^3} = 1$$

$$B_{min} = \left(\frac{Q^2}{gy_c^3}\right)^{1/3}$$

$$= \left(\frac{2^2}{9.81 \times 0.3197^3}\right)^{1/2}$$

$$= 3.5325\text{m} \simeq 3.53\text{m}$$

34. The consoliated data of a spot study for a certain stretch of a highway is given in the table.

Speed range (kmph)	Number of observations
0 – 10	7
10 – 20	31
20 – 30	76
30 – 40	129
40 – 50	104
50 – 60	78
60 – 70	29
70 – 80	24
80 – 90	13
90 – 100	9

The "upper speed limit" (in kmph) for the traffic sign is

(a) 70 (b) 55

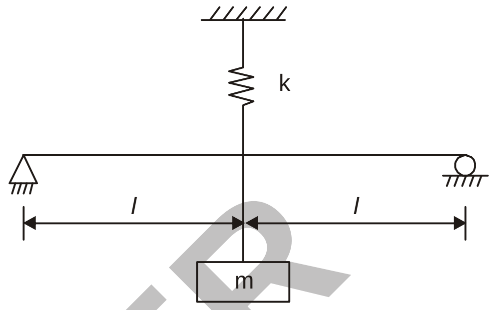
(c) 50 (d) 65

Sol. (b)

Mid speed (kmph)	% of vehicles	Cumulative %
5	1.4	1.4
15	6.2	7.6
25	15.2	22.8
35	25.8	48.6
45	20.8	69.4
55	15.6	85
65	5.8	90.8
75	4.8	95.6
85	2.6	98.2
95	1.8	100

Hence 85^{th} percentile speed or safe speed = 55 km/hr

35. A linearly elalstic beam of length $2l$ with flexural rigidity EI has neglitible mass. A massless spring with a spring constant k and a rigid block of mass m are attached to the beam as shown in the figure.



The natural frequency of this system is

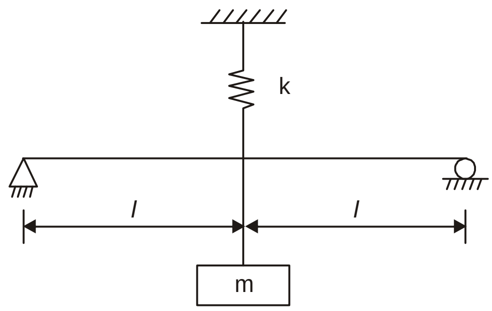
(a) $\sqrt{\dfrac{kl^3 + 6EI}{ml^3}}$

(b) $\sqrt{\dfrac{kl^3 + 48EI}{ml^3}}$

(c) $\sqrt{\dfrac{6EIk}{(kl^3 + 6EI)m}}$

(d) $\sqrt{\dfrac{48EIk}{(kl^3 + 48EI)m}}$

Sol. (a)



Let us consider sfittness of beam as k_b.

Here both the stiffness elements are in parallel.

$$\Rightarrow \quad k_{eq} = k_1 + k_2$$

$$= k + \frac{48EI}{(2\ell)^3} = k + \frac{6EI}{\ell^3}$$

Natural frequency $\omega_n = \sqrt{\dfrac{k_{eq}}{m}}$

$$= \sqrt{\frac{\left(k+\frac{6EI}{\ell^3}\right)}{m}} = \sqrt{\frac{k\ell^3+6EI}{m\ell^3}}$$

Hence, the correct option is (a)

36. Consider the statements P and Q related to the analysis/design of retaining walls.

P: When a rough retaining wall moves toward the backfill, the wall friction force/resistance mobilizes in upward direction along the wall.

Q: Most of the earth pressure theories calculate the earth pressure due to surcharge by neglecting the actual distribution of stresses due to surcharge.

Which of the following options is CORRECT?

(a) Both P and Q are true

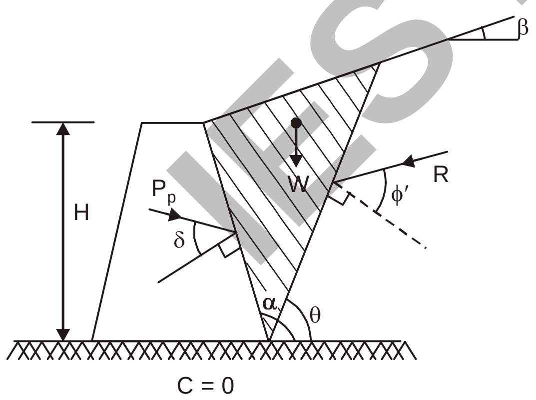
(b) P is true and Q is false

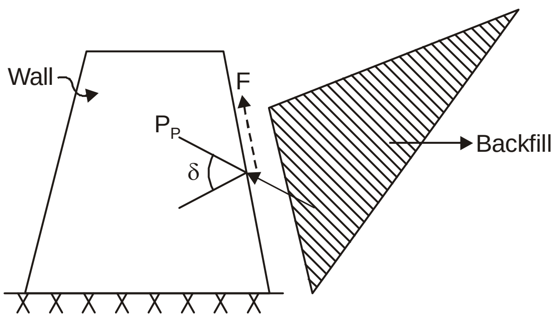
(c) P is false and Q is true

(d) Both P and Q are false

Sol. (a)

- When rough retaining wall moves toward the back fill passive earth pressure (P_P) condition will develop which can be represented with the help of diagram shown below.





- Hence wall friction force/resistance (F) mobilizes in upward direction along the wall.
- Earth pressure due to surcharge is assumed to be constant along the depth of retaining wall & actual variation of stress due to surcharge is neglected.

37. A circular settling tank is to be desinged for primary treatment of sewage of a flow rate of 10 million liters/day. Assume a detention period of 2.0 hours and surface loading rate of 40000 liters/m^2/day. The height (in meters) of the water column in the tank is ________. (rounded off to 2 decimal places)

Sol. (3.33)

Given,

Discharge, Q = 10 MLD

Detention period, t_d = 2 hr

Over flow rate, V_0 = 40,000 litres/m^2/day

H = ?

As, Surface area of tank = $\frac{\text{Discharge}}{\text{Over flow rate}} = \frac{Q}{V_0}$

So,

$$\text{S.A.} = \frac{10\times10^6\,\text{L/day}}{40{,}000\ \text{L/m}^2\text{/day}}$$

$$\boxed{\text{S.A} = 250\text{m}^2}$$

Volume of tank (V) = Q × t_d

$$= \frac{10\times10^6\times10^{-3}}{24}\times2\ \text{m}^3$$

$$\boxed{V = 833.33\text{m}^3}$$

Hence,

Height of water in tank or Height of setting zone $= \frac{V}{\text{S.A}}$

$$H = \frac{833.33\text{m}^3}{250\text{m}^2}$$

$$\boxed{H = 3.33\text{m}}$$

Note: In reality, height of water in tank OR height of settling zone is estimated by above approach but if the bottom of sedimentation tank is assumed to be sloping or hoppered (i.e sludge zone is also considered) then height of water in tank is determined as follows:

Using,

$$\boxed{\text{Volume of tank }(V) = D^2\left(\frac{\pi}{4}H + 0.011D\right)} \quad ....(1)$$

As,

$$\text{S.A.} = \frac{\pi}{4}D^2 = 250\text{m}^2$$

$$\boxed{D = 17.835\text{m}}$$

So, $833.33 = (17.835)^2\left(\frac{\pi}{4}H + 0.011(17.835)\right)$

So, $\boxed{H = 3.085\text{m}}$

38. Consider the statements P, Q and R.

P: Compacted fine-grained soils with flocculated structure have isotropic permeability.

Q: Phreatic surface/line is the line along which the pore water pressure is always maximum.

R: The piping phenomenon occuring below the dam foundation is typically known as blowout piping.

Which of the following option(s) is/are CORRECT?

(a) Both P and R are true

(b) Both Q and R are false

(c) P is false and Q is true

(d) P is true and R is false

Sol. **(b,d)**

- A fine grained soil when compacted on dry side of optimum has a flocculant (random) structure.
- Compacted fine grained soils with flocculant structure has isotropic permeability
- The top most flow line below which seepage takes place through a dam body is called phreatic line.

  Hydrostatic pressure is observed below phreatic line where as pressure on & above the phreatic line is atmospheric.

39. In the context of pavement material characterization, the CORRECT statement(s) is/are

(a) In compacted bituminous mix. voids in the mineral aggregate (VMA) is equal to the sum of total volume of air voids (V_v) and total volume of bitumen (V_b).

(b) The toughness and hardness of road aggregates are determined by Los Angeles abrasion test and aggregate impact test, respectively.

(c) The load penetration curve of CBR test may need origin correction due to the non-vertical penetrating plunger of the loading machine.

(d) Grading of normal (unmodified) bitumen binders is done based on viscosity test results.

Sol. **(a, c, d)**

- In a bituminous mix

  VMA = Voids in mineral aggregate

  VMA = V_b% + V_v%

  where, V_b% = Percentage volume of bitumen

  V_v% = Percentage air voids
- Toughness is determined by aggregate impact test and hardness is determined by los angeles abrasion test.
- An initial concavity in CBR curve indicate errors which may occur due to following reasons and also require correction

  (a) Top layer of soil too soft

  (b) Top surface of soil specimen is not even

  (c) The penetration plunger of the loading machine is not vertical and the bottom of plunger is not horizontal.

40. Consider two matrices

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}.$$

The determinant of the matrix AB is _______ (in integer).

Sol. (10)

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$$

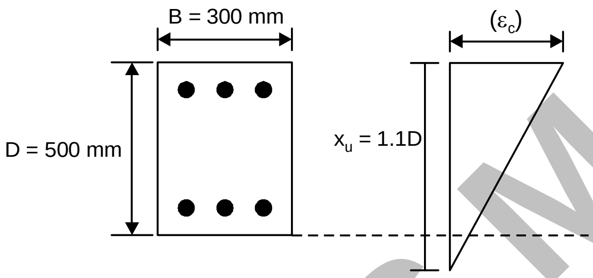
$$B = \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2+2+4 & 0+3+16 \\ -1+0+3 & 0+0+12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 19 \\ 2 & 12 \end{bmatrix}$$

$$= 48 - 38 = 10$$

41. A concrete column section of size 300mm×500mm as shown in the figure is subjected to both axial compression and bending along the major axis. The depth of the neutral axis (x_u) is 1.1 times the depth of the column, as shown.



Column Section Strain distribution

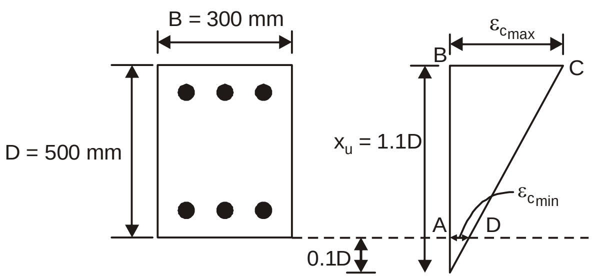
The maximum compressive strain (ε_c) at highly compressive extreme fiber in concrete, where there is no tension in the section, is ________$\times 10^{-3}$ (rounded off to 2 decimal places)

Sol. (3.28)

The minimum compressive strain in the column

$$\varepsilon_{c_1 max} = 0.0035 - 0.75\varepsilon_{c_1 min}$$

$\varepsilon_{c_1 min}$ = strain in the least compressed fibre



Column Section Strain distribution

In the strain diagram,

From triangle ΔOAD and ΔOBC

$$\Rightarrow \quad \frac{\varepsilon_{c_1 max}}{1.1D} = \frac{\varepsilon_{c_1 min}}{0.1D} \Rightarrow \varepsilon_{c_1 min} = \frac{1}{11}\varepsilon_{c_1 max}$$

$$\varepsilon_{c_1 max} = 0.0035 - 0.75 \times \frac{1}{11}\varepsilon_{c_1 max}$$

$$\Rightarrow \quad \varepsilon_{c_1 max} = 3.276 \times 10^{-3} = 3.28 \times 10^{-3}$$

Hence, the correct answer is 3.28

42. A vertical summit curve on a freight corridor is formed at the intersection of two gradients, +3.0% and –5.0%.

Assume the following:

Only large-sized trucks are allowed on this corridor.

Design speed = 80 kmph

Eye height of truck drivers above the road surface = 2.30m

Height of object above the road surface for which trucks need to stop = 0.35m

Total reaction time of the truck drivers = 2.0s

Coefficient of longitudinal friction of the road = 0.36

Stopping sight distance gets compesnated on the gradient.

The design length of the summit curve (in meters) to accommodate the stopping sight distance is _____ (rounded off to 2 decimal places).

Sol. (117.7)

$$\text{Sight distance (S)} = V \cdot t_a + \frac{V^2}{2g\mu}$$

$$= \left(\frac{5}{18} \times 80 \times 2\right) + \frac{\left(\frac{5}{18} \times 80\right)^2}{2 \times 9.8 \times 0.36}$$

$$= 44.44 + 69.92 = 114.36 \text{ m}$$

Assume L > S

$$L = \frac{NS^2}{2\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

$$= \frac{0.08 \times 1140.36^2}{2\left(\sqrt{2.3} + \sqrt{0.35}\right)^2} = 117.7$$

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43. An organic waste is represented as $C_{240}O_{200}H_{180}N_5S$

(Atomic weights: S-32, H-1, C-12, O-16, N-14)

Assume complete conversion of S to SO_2 while burning.

SO_2 generated (in grams) per kg of this waste is ________ (rounded off to 1 decimal place).

Sol. (10)

Organic waste = $C_{240}O_{200}H_{180}N_5S$

Weight of organic waste = 1 kg

Burning of organic waste converts

sulphur into sulphur dioxide as follows:

$$\underset{(6362\text{ g})}{C_{240}O_{200}H_{180}N_5S} + O_2 \rightarrow xCO_2 + yH_2O + zH_2O + \underset{(64\text{g})}{SO_2}$$

i.e. 6362 of organic waste produces 64g of sulphur dioxide.

So,

$$1\text{ kg of organic waste produces } \frac{64}{6362}\times 1000 \text{ gm of } SO_2$$

So, $\boxed{\text{Amount of } SO_2 \text{ produced} = 10\text{ grams/kg}}$

44. A hypothetical multimedia filter, consisting of anthracite particles (specific gravity: 1.50), silica sand (specific gravity: 2.60), and ilmenite sand (specific gravity: 4.20), is to be designed for treating water/wastewater. After backwashing, the particles should settle forming three layers: coarse anthracite particles at the top of the bed, silica sand in the middle, and small ilmenite sand particles at the bottom of the bed.

Assume

(i) Slow discrete settling (Stoke's law is applicable)

(ii) All particles are spherical

(iii) Diameter of silica sand particles is 0.20mm

The correct option fulfilling the diameter requirements for this filter media is

(a) diameter of anthracite particles is slightly less than 0.64 mm and diameter of ilmenite particles is slightly less than 0.10mm.

(b) diameter of anthracite is slightly greater than 0.64 mm and diameter of ilmenite particles is slightly than 0.10 mm

(c) diameter of anthracite is slightly greater than 0.35mm and diameter of ilmenite particles is slightly less than 0.141 mm.

(d) diameter of anthracite particles is slightly less than 0.35 mm and diameter of ilmenite particles is slightly greater than 0.141 mm

Sol. (d)

Anthracite $G_s = 1.50$
Silica sand $G_s = 2.60$ $D = 0.2\text{mm}$
Ilmenite sand $G_s = 4.20$

Let the settling velocity of anthracite pacticle at top, silica pacticle at middle & ilmenite pacticle at bottom, after back washing be $(V_S)_T$, (V_S), $(V_S)_B$ respectively.

For Middle silica sand

$$V_S = \frac{(G_{S_1}-1)\gamma_w D_1^2}{18\mu}$$

$$V_S = K(2.6-1)(0.2)^2$$

$\boxed{V_S = K(1.6)(0.2)^2}$ where, $K = \dfrac{\gamma_w}{18\mu}$

For Top Anthracite

$$(V_S)_T = K(G_{S_2}-1)(D_2)^2$$

$$(V_S)_T = K(0.5)(D_2)^2$$

As anthracite lies above silica layer

So, anthracite particles should have settling velocity less than silica particles

$$(V_S)_T < V_S$$

So, $K(0.5)(D_2)^2 < K(1.6)(0.2)^2$

$$D_2 \text{ or } D_{Top} < \sqrt{\frac{1.6\times(0.2)^2}{0.5}}$$

$\boxed{D_2 \text{ or } D_{Top} < 0.357\text{mm}}$

For bottom ilmenite sand

$$(V_s)_B = K(G_{s_3}-1)(D_3^2)$$

$$(V_s)_B = K(4.2-1)(D_3^2)$$

As, ilimite layer lies below silica lyers

So, ilimite particles should have settling velocity greater than silica

i.e., $(V_s)_B > V_s$

$$K(3.2)(D_3^2) > K(1.6)(0.2)^2$$

$$D_3 \text{or } D_{bottom} > \sqrt{\frac{1.6\times(0.2)^2}{3.2}}$$

$$\boxed{D_3 \text{ or } D_{bottom} > 0.141\text{mm}}$$

Hence, diameter of anthracite particle is slightly greater than 0.35mm & diameter of ilmenite particle is slightly greater than 0.141mm

So (d) is the correct option.

45. A storm with a recorded precipitation of 11.0 cm, as shown in the table, produced a direct run-off of 6.0cm

Time from start (hours)	1	2	3	4	5	6	7	8
Recorded cumulative precipitation (cm)	0.5	1.5	3.1	5.5	7.3	8.9	10.2	11.0

The ϕ-index of this storm is _______ cm/hr (rounded off to 2 decimal places)

Sol. (0.643)

Assume $\phi \le i_{lowest}$

$$\phi = \left(\frac{P-R}{t}\right)_{eff.\ duration}$$

$$= \frac{11-6}{8} = 0.625$$

Assume $0.625 < \phi \le 0.8$

$$\phi = \left(\frac{P-R}{t}\right)_{eff.}$$

$$= \frac{10.5-6}{7} = 0.643$$

t (m)	Rainfall (cm)	Intensity cm/h
0 – 1	0.5	0.5
1 – 2	1	1
2 – 3	1.6	1.6
3 – 4	2.4	2.4
4 – 5	1.8	1.8
5 – 6	1.6	1.6
6 – 7	1.3	1.3
7 – 8	0.8	0.8

46. A homogeneous earth dam has a maximum water head difference of 15m between the upstream and downstream sides. A flownet was drawn with the number of potential drops as 10 and the average length of the element as 3m. Specific gravity of the soil is 2.65. For a factor of safety of 2.0 against piping failure, void ratio of the soil is _______ (rounded off to 2 decimal places).

Sol. (0.65)

(ΔH) total head loss = 15m

(n_d) number of equipotential drops = 10

(l) length of flow field = 3m

(G) specific gravity = 2.65

(FOS) factor of safety = 2

(e) void ratio = ?

(Δh) equipotential drop of head

$$= \frac{\text{Total head loss}}{\text{no. of equipotential drop}}$$

$$\Delta h = \frac{\Delta H}{N_d} = \frac{15}{10} = 1.5\text{m}$$

FOS against quick sand condition = $\frac{i_{cr}}{i}$

(i) hydraulic gradient = $\frac{\Delta h}{l} = \frac{1.5}{3} = 0.5$

$2 = \frac{i_{cr}}{0.5}$, (i_{cr}) ciritical hydraulic gradient = 1

Critical hydraulic gradient $(i_{cr}) = \frac{G-1}{1+e} = 1 = \frac{2.65-1}{1+e}$

(e)void ratio = 0.65

47. A critical activity in a project is estimated to take 15 days to complete at a cost of Rs. 30,000. The activity can be expedited to complete in 12 days by spending a total amount of Rs. 54,000. Consider the statements P and Q.

P: It is economically advisable to complete the activity early by crashing if the indirect cost of the project is Rs. 8,500 per day.

Q: It is economically advisable to complete the activity early by crashing, if the indirect cost of the project is Rs. 10,000 per day.

Which one of the following options is CORRECT?

(a) P is true and Q is false

(b) Both P and Q is false

(c) P is false and Q is true

(d) Both P and Q are true

Sol. (d)

$$C/s = \frac{54000-30000}{15-12} = 8000\ \text{Rs / day}$$

$\Rightarrow$ If C/s is less than indirect cost per day then it would be economically advisable to complete the activity early by crashing. Hence both the statements P&Q are correct.

48. A child walks on a level surface from point P to point Q at a bearing of 30°, from point Q to point R at a bearing of 90° and then directly returns to the starting point P at a bearing of 240°. The straight-line paths PQ and QR are 4m each. Assuming that all bearings are measured from the magnetic north in degrees, the straight-line path length RP (in meters) is _______. (rounded off to the nearest integer)

Sol. (7m)

Line	Length	Bearing
PQ	4m	30°
QR	4m	90°
RP	L	240°

Closed traverse,

$$\Sigma L = 0,\quad 4\cos 30° + 4\cos 90° + L\cos 240° = 0$$

$$\therefore\quad L = 6.928\ \text{m} \approx 7\ \text{m}$$

49. A homogenous, prismatic, linearly elastic steel bar fixed at both the ends has a slenderness ratio (l/r) of 105, where l is the bar length and r is the radius of gyration. The coefficient of thermal expansion of steel is 12×10^{-6}/°C. Consider the effective length of the steel bar as $0.5l$ and neglect the self-wieght of the bar.

The differential increase in temperature (rounded off to the nearest integer) at which the bar buckles is

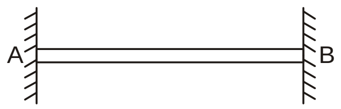
(a) 85°C

(b) 250°C

(c) 400°C

(d) 298°C

Sol. (d)



$$\sigma_{temp} = E\alpha\Delta T$$

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda_{eff}^2} = \frac{\pi^2 E}{(\ell_{eff}/r)^2}$$

$\Rightarrow$ For given support $\rightarrow \ell_{eff} = \dfrac{l}{2}$

Hence,

$$E\alpha\Delta T = \frac{\pi^2 E}{\left(\frac{l}{2r}\right)^2}$$

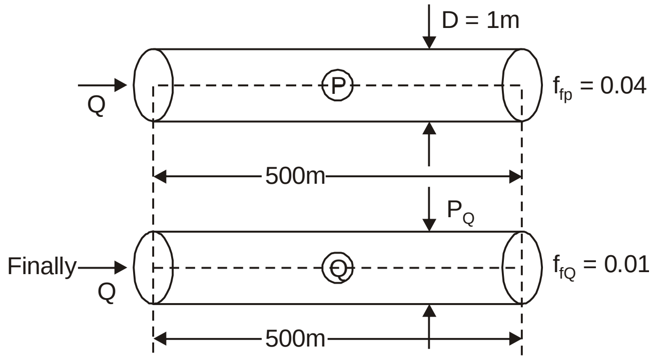
$$\Rightarrow\quad \Delta T = \frac{\pi^2\times4}{\alpha5\left(\frac{l}{2}\right)^2} = \frac{\pi^2\times4}{12\times10^{-6}\times105^2}$$

$$\Rightarrow\quad \Delta T = 298°C$$

50. A 500 m long water distribution pipeline P with diameter 1.0m, is used to convey 0.1m^3/s of flow. A new pipeline Q, with the same length and flow rate, is to replace P. The friction factors for P and Q are 0.04 and 0.01, respectively. The diameter of the pipeline Q (in meters) is ________ (rounded off to 2 decimal places)

Sol. (0.76)

Initially, we have,



To replace the pipe, head loss has to be same

$$h_{fp} = h_{fQ}$$

$$\Rightarrow \quad \frac{f_{fp}LQ^2}{12.1D_P^5} = \frac{f_{fQ}LQ^2}{12.1D_Q^5}$$

$$\Rightarrow \quad \frac{0.04 \times 500 \times Q^2}{12.1 \times 1^5} = \frac{0.01 \times 500 \times Q^2}{12.1 \times D_Q^5}$$

$$\Rightarrow \quad D_Q^5 = \frac{0.01}{0.04}$$

$$\Rightarrow \quad D_Q = \left(\frac{0.01}{0.04}\right)^{1/5} = 0.7579\text{m}$$

$$\simeq 0.76\text{m}$$

51. A horizontal curve of radius 1080m (with transition curves on either side) in a Broad Gauge railway track is designed and constructed for an equilibrium speed of 70 kmph. However, a few years after construction, the Railway Authorities decided to run express trains on this track. The maxium allowable cant deficiency is 10cm.

The maximum restricted speed (in kmph) of the express trains running on this track is _______ (rounded off to the nearest integer)

Sol. (113 kmph)

Constructed equillibrium cant

$$= \frac{GV_{eq}^2}{127R} = \frac{1.75 \times 70^2}{127 \times 1080} = 0.062518 \text{ m} = 62.52 \text{ mm}$$

With allowable cant deficiency of 100 mm

Maximum speed of train can run with theoretical cant = 62.52 + 100 = 162.52 mm

$$\therefore \quad 162.52 \times 10^{-3} = \frac{1.75 \times V_m^2}{127 \times 1080}$$

$$\therefore \quad V_m = 112.86 \text{ kmph} \simeq 113 \text{ kmph}$$

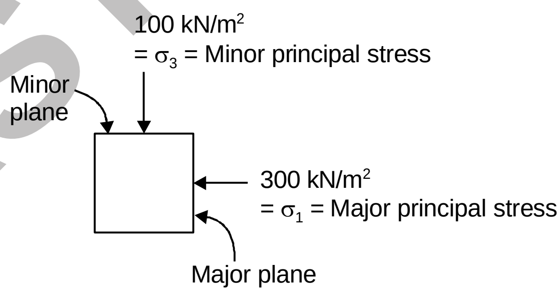
52. A drained triaxial test was conducted on a saturated sand specimen using a stress-path triaxial testing system. The specimen failed when the axial stress reached a value of 100 kN/m^2 from an initial confining pressure of 300 kN/m^2.

The angle of shearing plane (in degrees) with respect to horizontal is _______ (rounded off to the nearest integer).

Sol. (30°)

In stress-path triaxial testing, we can control both radial and axial stress.

Failure situation



$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2C\tan\left(45° + \frac{\phi}{2}\right)$$

$$300 = 100\tan^2\left(45° + \frac{\phi}{2}\right) + 0$$

[Since for drained test C = 0]

$$\Rightarrow \quad \phi = 30°$$

Angle between normal to major plane and failure plane is $45 + \frac{\phi}{2}$

Angle between normal to minor plane and failure plane is $45° - \frac{\phi}{2}$

$\Rightarrow$ Angle between minor plane and failure plane

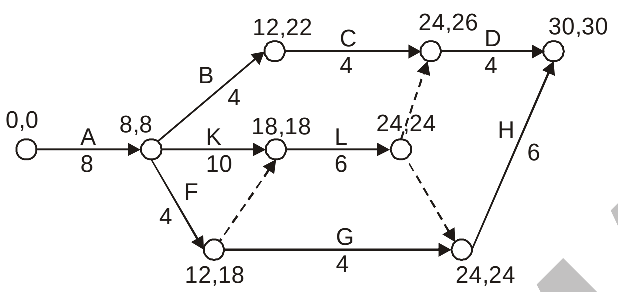
$$= 45 - \frac{\phi}{2} = 45° - \frac{30°}{2} = 30°$$

53. The table shows the activities and their durations and dependencies in a project.

Activity	Duration(days)	Depends on
A	8	–
B	4	A
C	4	B
D	4	C,L
F	4	A
G	4	F
H	6	G,L
K	10	A
L	6	F,K

The total duration (in days) of the project is _____(in integer)

Sol. (30)



Critical path:

A-K-L-H

Project duration = 30 days

54. The in-situ percentage of voids of a sand deposit is 50%. The maximum and minimum densities of sand determined from the laboratory tests are 1.8 g/cm^3 and 1.3 g/cm^3, respectively. Assume the specific gravity of sand as 2.7.

The relative density index of the in-situ sand is _____ (rounded off to 2 decimal places)

Sol. (0.13)

maximum density $(\gamma_d)_{max} = 1.8\,g/cm^3$

minimum density $(\gamma_d)_{min} = 1.3\,g/cm^3$

specific gravity (G) = 2.7

(I_D) relative density = $\dfrac{\gamma_{dmax}(\gamma_d - \gamma_{dmin})}{\gamma_d(\gamma_{dmax} - \gamma_{dmin})}$

$\therefore$ (n) percentage of voids

$$= \frac{(V_v)\text{volume of voids}\times 100}{(V_T)\text{Total volume}} = 50\%$$

Porosity(n) = 0.5

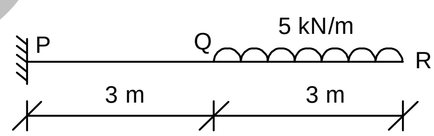
void ratio (e) = $\dfrac{n}{1-n} = \dfrac{0.5}{1-0.5} = 1$

$(\gamma_d)_{insitu} = \dfrac{G\cdot\gamma_w}{1+e} = \dfrac{2.7\times 1}{1+1} = 1.35\,g/cm^3$

(I_D) = $\dfrac{1.8(1.35-1.3)}{1.35(1.8-1.3)} = 0.13$

Relative density (I_D) = 0.13

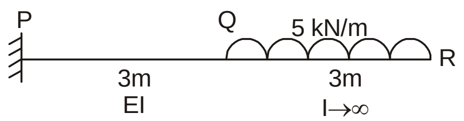
55. For the 6m long horizontal cantilever beam PQR shown in the figure. Q is the mid-point. Segment PQ of the beam has flexural rigidity EI = 2 × 10^5 kN. m^2 whereas the segment QR has infinite flexural rigidity. Segment QR is subjected to uniformly distributed, vertically downward load of 5 kN/m.

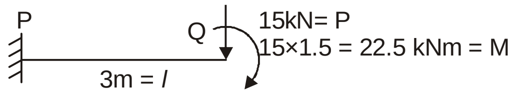


(Figure NOT to scale)

The magnitude of the vertical displacement (in mm) at point Q is _______ (rounded off to 3 decimal places)

Sol. (1.181mm)





Vertical deflection at Q = $\dfrac{Pl^3}{3EI} + \dfrac{Ml^2}{2EI}$

$$\Delta_Q = \frac{(15kN)(3m)^3}{3\times 2\times 10^5 kNm^2} + \frac{(22.5kNm)}{2\times 2\times 10^5}$$

$$\Delta_Q = (6.75\times 10^{-4} + 5.0625\times 10^{-4})m$$

$$\Delta_Q = 1.181mm$$