IES MASTER
Institute for Engineers (IES/GATE/PSUs)

GATE 2024

CIVIL ENGINEERING

Detailed Solution

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The colour of the Questions show the difficulty level of questions as per below mentioned colour code:

- Easy
- Moderate
- Hard

SECTION: GENERAL APTITUDE

1. On a given day, how many times will the second-hand and the minute-hand of a clock cross each other during the clock time 12:05:00 hours to 12:55:00 hours?

(a) 49 (b) 50

(c) 55 (d) 51

Sol. (b)

Minute hand covers 6° angle in 1 min.

Second hand covers 360° angle in 1 min.

Means 354° angle gained by second hand over min. hand in 1 min. of time

So, 360° angle gained in $\frac{360}{354}$ min.

(which is required time for one crossing)

But first crossing taking $\left(\frac{30}{354}\right)$ min. only

So, total time taken in 50

Crossings = $1\times\frac{30}{354}+49\times\frac{360}{354}=49.91$ min.

So, no. of crossings = 50

2. If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [simmer $\rightarrow$ seethe $\rightarrow$ smolder] is analogous to [break $\rightarrow$ raze $\rightarrow$ ___] Which one of the given options is appropriate to fill the blank?

(a) fissure (b) obliterate

(c) fracture (d) obfuscate

Sol. (b)

Meaning of the words

simmer $\rightarrow$ seethe $\rightarrow$ smolder

is analogous to

brake $\rightarrow$ raze $\rightarrow$ obliterate

3. In a locality, the house are numbered in the following way:

The house-numbers on one side of a road are consecutive odd integers starting from 301, while the house-numbers on the other side of the road are consecutive even numbers starting from 302. The total number of houses is the same on both sides of the road.

If the difference of the sum of the house-numbers between the two sides of the road is 27, then the number of houses on each side of the road is

(a) 52 (b) 54

(c) 27 (d) 26

Sol. (c)

Let no. of houses on each side = n

Sum of odd numbered houses

= (301 + 303 + 305, …, n)

$= \frac{n}{2}[2\times301+(n-1)2]$

$= \frac{n}{2}[2n+600]=n(n+300)$

Sum of even numbered houses

= 302 + 304 + 306, …, n

$= \frac{n}{2}[2\times302+(n-1)2]$

$= \frac{n}{2}[2n+602]$

= n[n + 301]

According to question

n[n + 301] – n[n + 300] = 27

n[n + 30] – n – 300] = 27

n = 27

n[n + 301] – n[n + 300] = 27

n[n + 30] – n – 300] = 27

n = 27

Alternate method:

If one house each side difference = 302 – 301 = 1

If two house each side difference

= (302 + 304) – (301 + 303) = 2

If three houses each side difference

= (302 + 304 + 306) – (301 + 303 + 39) = 3

So, to make difference equal to 27 number of houses on each side must be 27.

4. Which one of the given options is a possible value of x in the following sequence?

3, 7, 15, x, 63, 127, 255

(a) 31 (b) 35

(c) 45 (d) 40

Sol. (a)

$7 - 3 = 4 = 4 \times 1$

$15 - 7 = 8 = 4 \times 2$

$\boxed{x - 15 = 16 = 8 \times 2}$

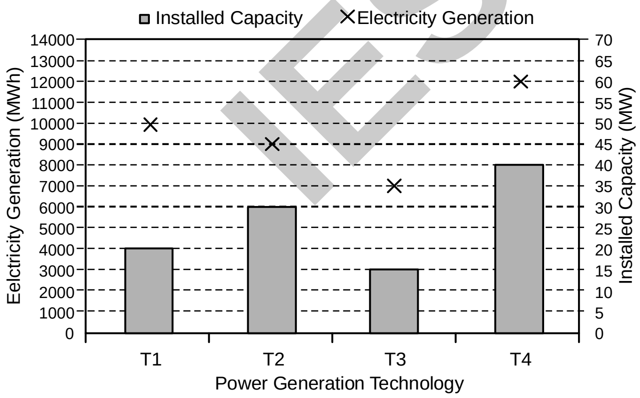
$63 - x = 32 = 16 \times 2$

$127 - 63 = 64 = 32 \times 2$

$255 - 127 = 128 = 64 \times 2$

$x - 15 = 16 \Rightarrow x = 31$

5. The chart given below compares the Installed Capacity (MW) of four power generation technologies, T1, T2, T3 and T4 and their Electricity Generation (MWh) in a time of 1000 hours (h).



The capacity factor of a power generation technology is:

$$\text{Capacity factor} = \frac{\text{Electricity Generation (MWh)}}{\text{Installed Capacity (MW)} \times 1000\text{ (h)}}$$

Which one of the given technologies has the highest Capacity Factor?

(a) T3 (b) T4

(c) T2 (d) T1

Sol. (d)

$$\text{Capacity factor of T1} = \left(\frac{10000}{20}\right) \times \frac{1}{1000} = 0.5$$

$$\text{Capacity factor of T2} = \left(\frac{9000}{30}\right) \times \frac{1}{1000} = 0.3$$

$$\text{Capacity factor of T3} = \left(\frac{7000}{15}\right) \times \frac{1}{1000} = 0.47$$

$$\text{Capacity factor of T4} = \left(\frac{12000}{40}\right) \times \frac{1}{1000} = 0.3$$

So, the capacity of power generation technology T1 (0.5) is highest.

6. For positive integers p and q, with

$\frac{p}{q} \neq 1, \left(\frac{p}{q}\right)^{\frac{p}{q}} = p^{\left(\frac{p}{q} - 1\right)}$. Then,

(a) $\sqrt{q} = \sqrt{p}$ (b) $\sqrt[p]{q} = \sqrt[q]{p}$

(c) $q^p = p^{2q}$ (d) $q^p = p^q$

Sol. (d)

$$\left(\frac{p}{q}\right)^{p/q} = p^{\left(\frac{p}{q} - 1\right)}$$

$$\frac{p^{p/q}}{q^{p/q}} = p^{p/q} \times p^{-1}$$

$$\left(\frac{p^{p/q}}{q^{p/q}} - \frac{p^{p/q}}{p}\right) = 0$$

$$p^{p/q}\left(\frac{1}{q^{p/q}} - \frac{1}{p}\right) = 0$$

$$\left(\frac{1}{q^{p/q}} - \frac{1}{p}\right) = 0 \qquad \left(\because\ p^{p/q} \neq 0\right)$$

$$\frac{p - q^{p/q}}{p \cdot q^{p/q}} = 0$$

$\Rightarrow p = q^{p/q}$

$p = (q^{1/q})^p$

$p^{1/p} = q^{1/q}$

$q^p = p^q$

7. Three distinct sets of indistinguishable twins are to be seated at a circular table that has 8 identical chairs. Unique seating arrangements are defined by the relative positions of the people.

How many unique seating arrangements are possible such that each person is sitting next to their twin?

(a) 28 (b) 14

(c) 10 (d) 12

Sol. (d)

8. In the 4 × 4 array shown below, each cell of the first three columns has either a cross (X) or a number, as per the given rule.

1	1	2	
2	X	3	
2	X	4	
1	2	X	

Rule: The number in a cell represents the count of crosses around its immediate neighboring cells (left, right, top, bottom, diagonals).

As per this rule, the maximum number of crosses possible in the empty column is

(a) 0 (b) 1

(c) 3 (d) 2

Sol. (d)

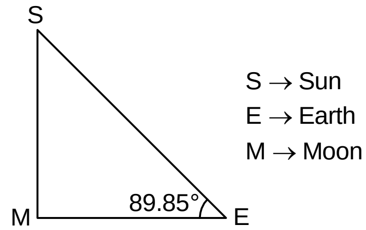
1	1	2	X
2	X	3	1
2	X	4	2
1	2	X	X

9. During a half-moon phase, the Earth-Moon-Sun from a right triangle. If the Moon-Earth-Sun angle at this half-moon phase is measured to be 89.85°, the ratio of the Earth-Sun and Earth-Moon distances is closest to

(a) 328 (b) 238

(c) 382 (d) 283

Sol. (c)



The distance from the earth to the moon

= 240000 mile (estimated)

$$\cos 89.85 = \frac{240000}{SE}$$

SE = 91673351.94 mile

$$\frac{ES}{EM} = \left(\frac{91673351.94}{240000}\right) = 381.97 = 382$$

10. In the given text, the blanks are numbered (i)–(iv). Select the best match for all the blanks.

From the ancient Athenian arena to the modern Olympic stadiums, athletics __(i)__ the potential for a spectacle. The crowd __(ii)__ with bated breath as the Olympian artist twists his body, stretching the javelin behind him. Twelve strides in, he begins to cross-step. Six cross-steps __(iii)__ in an abrupt stop on his left foot. As his body __(iv)__ like a door turning on a hinge, the javelin is launched skyward at a precise angle.

(a) (i) holds (ii) waits (iii) culminate (iv) pivots

(b) (i) hold (ii) wait (iii) culminate (iv) pivots

(c) (i) hold (ii) waits (iii) culminates (iv) pivot

(d) (i) holds (ii) wait (iii) culminates (iv) pivot

Sol. (a)

According to subject verb agreement,

singular noun takes singular verb and plural noun takes plural verb.

"athletics" is placed as a game (singular common noun) so singular verb form is used.

'crowd" is a singular noun, so singular verb form is used.

"cross-steps" is a plural form of noun, so it will be followed with a plural form of verb.

"his body" is a singular noun so singular form of verb is used.

Hence, option a is correct.

SECTION: CIVIL ENGINEERING

1. If the number of sides resulting in a closed traverse is increased from three to four, the sum of the interior angles increases by

(a) 360° (b) 180°

(c) 270° (d) 90°

Sol. (b)

Sum of interior angle = (2n – 4) × 90

For 3 side sum is 180°

For 4 side sum is 360°

∴ Sum increased by 180°

2. The second-order differential equation in an unknown function u: u(x, y) is defined as

$$\frac{\partial^2 u}{\partial x^2} = 2$$

Assuming g: g(x), f: f(y) and h: h(y), the general solution of the above differential equation is

(a) $u = x^2 + x\,f(y) + g(x)$

(b) $u = x^2 + f(y) + y\,g(x)$

(c) $u = x^2 + f(y) + g(x)$

(d) $u = x^2 + x\,f(y) + h(y)$

Sol. (d)

$$\frac{\partial^2 u}{\partial x^2} = 2 \quad \text{...(i)}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = 2$$

Integrating both side with respect to x

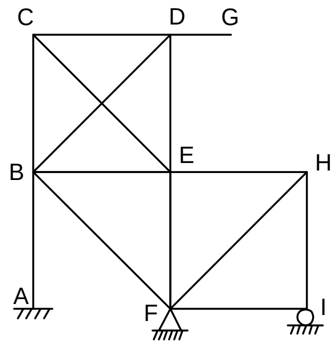
$$\frac{\partial u}{\partial x} = 2x + f(y)$$

Again integrating with respect to x

$$u = x^2 + xf(y) + f(y)$$

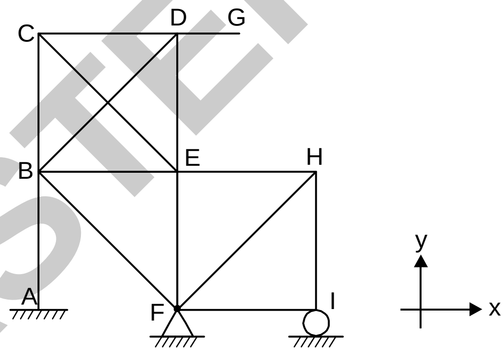
$$u = x^2 + xf(y) + h(y)$$

3. The plane frame shown in the figure has fixed support at joint A, hinge support at joint F, and roller support at joint I. In the figure, A to I indicate joints of the frame.



If the axial deformations are neglected, the degree of kinematic indeterminacy is ________ (in integer).

Sol. (13)



From inextensibility of BF & BA,

$$\Delta B_x = 0 \quad \Delta B_y = 0$$

From inextensibility of BE and EF,

$$\Delta E_x = 0 \quad \Delta E_y = 0$$

From inextensibility of CE and BC

$$\Delta C_x = 0 \quad \Delta C_y = 0$$

From inextensibility of BD and CD,

$$\Delta D_x = 0 \quad \Delta D_y = 0$$

From inextensibility of DG,

$$\Delta G_x = 0$$

From inextensibility of EH and HF,

$$\Delta H_x = 0 \quad \Delta H_y = 0$$

From inextensibility FI,

$$\Delta I_x = 0$$

$\Rightarrow$ If at joint (F), FB, FE, FH, FI are rigidly connected then possible displacements at F = θ_F.

If at joint I, FI and IH are rigidly connected, possible displacements at I = θ_I.

Hence unknown joint displacements are

$\theta_B, \theta_C, \theta_D, \theta_E, \theta_F, \theta_G, \Delta_{Gy}, \theta_H, \theta_I$

$\Rightarrow$ $\boxed{D_k = 9}$

Note: However, if somebody assumes all members at joint (F) to be connected with pin then at (F) we have unknown joint displacements as $\theta_{FB}, \theta_{FE}, \theta_{FH}, \theta_{FI}$.

Similarly, if at joint I if somebody assumes FI and HI to be pin connected then unknown joint displacements at I are θ_{IF}, θ_{IH}.

Hence, D_k will increase by 4.

$\Rightarrow$ D_k = 9 + 4 = 13

4. The number of degrees of freedom for a natural open channel flow with a mobile bed is

(a) 3 (b) 5

(c) 2 (d) 4

Sol. (d)

- Mobile boundary channels are those in which boundaries undergo deformation due to continous process of erosion and deposition due to flow.
- In mobile channels, depth of flow, bed width, bed slopes and layout changes with space and time.
- Hence, mobile boundary channels have 4 degree of freedom.

Note: In rigid boundary channel only depth of flow may vary with space and time depending upon nature of flow. Hence, rigid boundary channels have 1 degree of freedom.

5. Which one of the following statements related to bitumen is FALSE?

(a) Ductility test is carried out on bitumen to test its adhesive property and ability to stretch.

(b) Flash point of bitumen is the lowest temperature at which application of a test flame causes vapours of the bitumen to catch an instant fire in the form of flash under specified test conditions.

(c) Kinematic viscosity is a measure of resistance to the flow of molten bitumen under gravity.

(d) Softer grade bitumen possesses higher softening point than hard grade bitumen.

Sol. (d)

6. The elements that DO NOT increase the strength of structural steel are

(a) Carbon (b) Manganese

(c) Chlorine (d) Sulphur

Sol. (c, d)

Increase in carbon content increases the strength of steel. Also, manganese increases the strength of structural steel.

However, chlorine and sulphur do not increase the strength of structural steel.

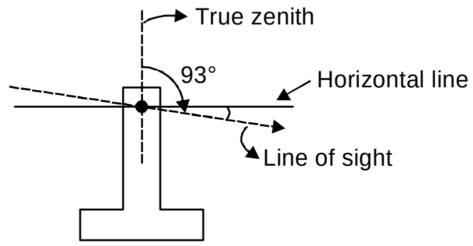
Hence, the correct option (c) and (d).

7. A surveyor observes a zenith angle of $93°00'00''$ during a theodolite survey. The corresponding vertical angle is

(a) $+87°00'00''$ (b) $-03°00'00''$

(c) $+03°00'00''$ (d) $-87°00'00''$

Sol. (b)



8. A car is travelling at a speed of 60 km/hr on a section of a National Highway having a downward gradient of 2%. The driver of the car suddenly observes a stopped vehicle on the car path at a distance 130 m ahead, and applies brake. If the brake efficiency is 60%, coefficient of friction is 0.7, driver's reaction time is 2.5 s, and acceleration due to gravity is 9.81 m/s^2, the distance (in meters) required by the driver to bring the car to a safe stop lies in the range

(a) 75 to 79 (b) 33 to 37

(c) 41 to 45 (d) 126 to 130

Sol. (a)

$$SSD = vt_r + \frac{v^2}{2g(\mu - 0.01n)}$$

$$= \left(\frac{5}{18}\times 60\right)\times 2.5 + \frac{\left(\frac{5}{18}\times 60\right)^2}{2\times 9.81(0.6\times 0.7 - 0.01\times 2)}$$

= 41.67 + 35.4

= 77.07

9. The smallest positive root of the equation

$x^5 - 5x^4 - 10x^3 + 50x^2 + 9x - 45 = 0$

lies in the range

(a) $6 \le x \le 8$ (b) $10 \le x \le 100$

(c) $2 < x \le 4$ (d) $0 < x \le 2$

Sol. (d)

f(0) = 0 – 45 = Negative

f(2) = $2^5 - 5 \times 2^4 - 10 \times 2^3 + 50 \times 2^2 + 9 \times 2 - 45$

= 32 – 65 – 80 + 200 + 18 – 45

= 60 = Positive

If f(x) is continuous on [a, b] and f(a) and f(b) are of opposite sign then there exist atleast one point c on [a, b] such that f(c) = 0 so atleast one root lying between (0, 2).

10. For the following partial differential equation,

$$x\frac{\partial^2 f}{\partial x^2} + y\frac{\partial^2 f}{\partial y^2} = \frac{x^2+y^2}{2}$$

Which of the following option(s) is/are CORRECT?

(a) hyperbolic for x < 0 and y > 0

(b) elliptic for x > 0 and y > 0

(c) elliptic for x = 0 and y > 0

(d) parabolic for x > 0 and y > 0

Sol. (a, b)

The most general case of second-order partial differential equation

$$A\frac{\partial^2 u}{\partial x} + B\frac{\partial^2 u}{\partial x\partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + fu = G \quad \ldots(i)$$

Given DE is

$$x\frac{\partial^2 f}{\partial x^2} + y\frac{\partial^2 f}{\partial y^2} = \frac{x^2+y^2}{2} \quad \ldots(ii)$$

Comparing (i) and (ii)

A = x and B = 0, C = y

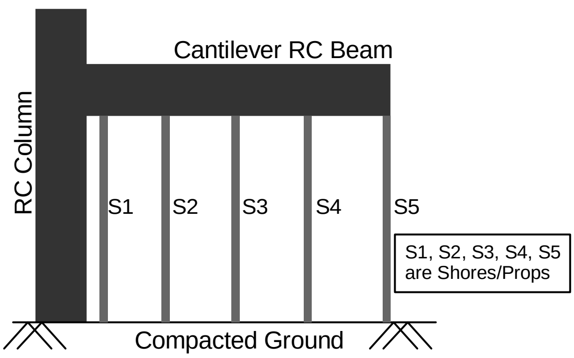
Discramanant = $B^2 - 4AC = 0 - 4 \times x \times y = -4xy$

$-4yx > 0$ if x < 0 and y > 0

So, given equation is hyperbolic if x < 0 and y > 0.

Similarly, we can prove if x > 0, y > 0 given PDE is elliptic also.

11. The following figure shows the arrangement of formwork for casting a cantilever RC beam.



The correct sequence of removing the Shores/Props is

(a) S3 → S4 → S2 → S5 → S1

(b) S1 → S2 → S3 → S4 → S5

(c) S3 → S2 → S4 → S1 → S5

(d) S5 → S4 → S3 → S2 → S1

Sol. (d)

While removing the props, we need to ensure that the beam type does not change i.e. if it is a cantilever beam then the props shall be removed in such a manner that the beam at the intermediate stage also shows cantilever behaviour.

If we remove prop 'S1' first then tensile stress will be generated at the bottom and no structural reinforcement is there to take care of this tension.

So, correct order is S5 → S4 → S3 → S2 → S1

Hence, the correct option is (d).

12. Consider the statements P and Q.

P: Soil particles formed by mechanical weathering and close to their origin are generally subrounded.

Q: Activity of the clay physically signifies its swell potential.

Which one of the following options is CORRECT?

(a) P IS FALSE AND Q is TRUE

(b) Both P and Q are TRUE

(c) P is TRUE and Q is FALSE

(d) Both P and Q are FALSE

Sol. (a)

- Bulky particles are formed mostly by mechanical weathering of rocks and minerals. Geologists use such terms as angular, subangular, subrounded and rounded to describe the shape of bulky particles. Small sand particles located close to their origin are generally very angular sand particles carried by wind and water for a long distance can be subangular to rounded in shape.
- Activity number of clay is used to study the swelling behaviour.

  Activity number (A_C)

$$= \frac{\text{Plasticity index}}{\text{\% of clay sized particles in soil}}$$

$A_C < 0.75 \rightarrow$ inactive

$0.75 < A_C < 1.25 \rightarrow$ normal active

$A_C > 1.25 \rightarrow$ active

Active means more prone to volume change.

13. Consider the data of f(x) given in the table.

i	0	1	2
x_i	1	2	3
$f(x_i)$	0	0.3010	0.4771

The value of f(1.5) estimated using second-order Newton's interpolation formula is ______ (rounded off to 2 decimal places).

Sol. (0.18)

$x_0 = 1, \; y_0 = 0$

$$\rightarrow \frac{y_1 - y_0}{x_1 - x_0} = 0.3010$$

$$\frac{y_{(1,2)} - y_{(0,1)}}{x_2 - x_1} = -0.1249$$

$x_1 = 2, \; y_1 = 0.3010$

$$\rightarrow \frac{y_2 - y_1}{x_2 - x_1} = 0.1761$$

$f(x) = y_0 + y_{(0,1)}(x - x_0) + y_{(0,12)}(x - x_0)(x - x_1)$

$f(x) = 0 + 0.3010(x - 1) + (-0.1249) \times (x - 1)(x - 2)$

$f(x) = 0.3010(x - 1) - 0.1249(x - 1)(x - 2)$

$f(1.5) = 0.3010 \times (1.5 - 1) - 0.1249(1.5 - 1)(1.5 - 2)$

$= 0.1505 + 0.03125 = 0.1817 \simeq 0.18$

14. Among the following statements relating the fundamental lines of a transit theodolite, which one is CORRECT?

(a) The Vernier of vertical circle must read zero when the line of collimation is vertical.

(b) The axis of plate level must lie in a plane parallel to the vertical axis.

(c) The line of collimation must be perpendicular to the horizontal axis at its intersection with the vertical axis.

(d) The axis of altitude level must be perpendicular to the line of collimation.

Sol. (c)

Line of collimation must be perpendicular to horizontal axis.

15. Consider the statements P and Q.

P: Client's preliminary estimate is used for budgeting costs toward the end of planning and design phase.

Q: Client's detailed estimate is used for controlling costs during the execution of the project.

Which one of the following options is CORRECT?

(a) Both P and Q are TRUE

(b) P is FALSE and Q is TRUE

(c) Both P and Q are FALSE

(d) P is TRUE and Q is FALSE

Sol. (a)

Various estimates used in construction industry and their purposes:

Estimate type	Purpose
Project proposal indicative cost estimate	Feasibility stage
Preliminary estimate	Budgeting cost towards end of planning and design phase
Detailed estimate	Controlling costs during the execution of project
Definitive estimate	To asses cost at completion
Final closure cost estimate	Final cost

Both the statements are correct.

Hence, the correct option is (a).

16. An embankment is constructed with soil by maintaining the degree of saturation as 75% during compaction. The specific gravity of soil is 2.68 and the moisture content is 17% during compaction. Consider the unit weight of water as 10 kN/m^3. The dry unit weight (in kN/m^3) of the compacted soil is __________ (rounded off to 2 decimal places).

Sol. (16.68)

Given,

Degree of saturation (S) = 75%

Specific gravity of soil (G) = 2.68

Moisture content (W) = 17%

Unit weight of water (γ_w) = 10 kN/m^3

Dry unit weight (γ_d) = ?

$\therefore$ Se = WG

0.75 × e = 0.17×2.68

(e) void ratio = 0.607

We know that

$$\gamma_d = \frac{G \cdot \gamma_w}{1+e} = \frac{2.68 \times 10}{1+0.607}$$

$$\boxed{(\gamma_d)\text{Dry unit weight} = 16.68\,\text{kN/m}^3}$$

17. The three-dimensional state of stress at a point is given by

$$\sigma = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{MPa}$$

The maximum shear stress at the point is

(a) 5 MPa (b) 20 MPa

(c) 25 MPa (d) 15 MPa

Sol. (b)

$$\sigma = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\sigma_1 = 40, \sigma_2 = 10, \sigma_3 = 0$$

$$\sigma_{max,abs} = \max\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right\}$$

$$\sigma_{max,abs} = \max\left\{\left|\frac{40-10}{2}\right|, \left|\frac{10-0}{2}\right|, \left|\frac{0-40}{2}\right|\right\}$$

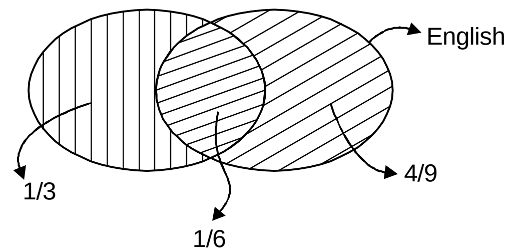
$$\tau_{max,abs} = 20 \text{ MPa}$$

18. The probability that a student passes only in Mathematics is 1/3. The probability that the student passes only in English is 4/9. The probability that the student passes in both of these subjects is 1/6. The probability that the student will pass in at least one of these two subjects is

(a) $\frac{1}{18}$ (b) $\frac{11}{18}$

(c) $\frac{17}{18}$ (d) $\frac{14}{18}$

Sol. (c)



$P(A\cup B\cup C)\rightarrow$ Probability that the student passes is atleast one subject.

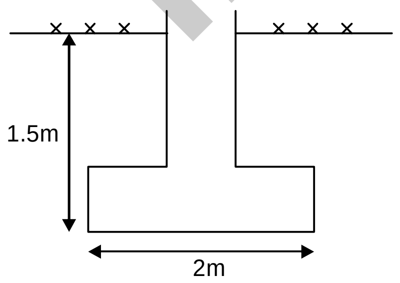
$$P(A\cup B\cup C) = \frac{1}{3}+\frac{1}{6}+\frac{4}{9}=\frac{6+3+8}{18}=\left(\frac{17}{18}\right)$$

19. A 2 m wide strip footing is founded at a depth of 1.5 m below the ground level in a homogeneous pure clay bed. The clay bed has unit cohesion of 40 kPa. Due to seasonal fluctuations of water table from peak summer to peak monsoon period, the net ultimate bearing capacity of the footing, as per Terzaghi's theory will

(a) increase (b) decrease

(c) remain the same (d) become zero

Sol. (c)



For pure clay, $\phi = 0$

N_c = 5.7, N_q = 1, $N_\gamma = 0$

Ultimate bearing capacity (q_u) = 5.7C + q

Net ultimate bearing capacity (q_{nu})

= 5.7C + q – q = 5.7C

Since cohesion is given to be constant, net ultimate bearing capacity will not change with water table fluctuations.

20. The primary air pollutant(s) is/are

(a) Lead (b) Sulphur dioxide

(c) Sulphuric acid (d) Ozone

Sol. (a, b)

(a) Primary air pollutants

(i) Particulate matter (PM)

(ii) Oxides of sulphur (SO_x)

(iii) Oxides of Nitrogen (NO_x)

(iv) Oxides of carbon (CO_x)

(v) Hydrocarbons (HC)

(vi) Volatile organic carbon (VOCs)

(b) Secondary air pollutants

(i) Ozone

(ii) Formaldehyde

(iii) PAN (peroxy acetyl nitrate)

(iv) Photo chemical smog

(v) Acid rain

→ Lead is a particulate matter (PM) and its main source in atmosphere is automobiles.

→ SO_2 is first abundant atmospheric contaminant in many cities. It is produced by chemical interaction between sulphur and oxygen.

21. As per the International Civil Aviation Organization (ICAO), the basic runway length is increased by x(%) for every y (m) raise in elevation from the Mean Sea Level (MSL). The values of x and y, respectively, are

(a) 5% and 200 m (b) 7% and 300 m

(c) 10% and 1000 m (d) 4% and 500 m

Sol. (b)

As per ICAO, basic runway length must be increased by 7% for 300 m elevation from MSL.

22. A 30 cm diameter well fully penetrates an unconfined aquifer of saturated thickness 20 m with hydraulic conductivity of 10 m/day. Under the steady pumping rate for a long time, the drawdowns in two observation wells located at 10 m and 100 m from the pumping well are 5 m and 1 m, respectively. The corresponding pumping rate (in m^3/day) from the well is ______ (rounded off to 2 decimal places).

Sol. (1854.61)

Given

Radius of well (r_w) = 15 cm

Hydraulic conductivity (k) = 10 m/day

(r_1) distance of first observation well = 10m

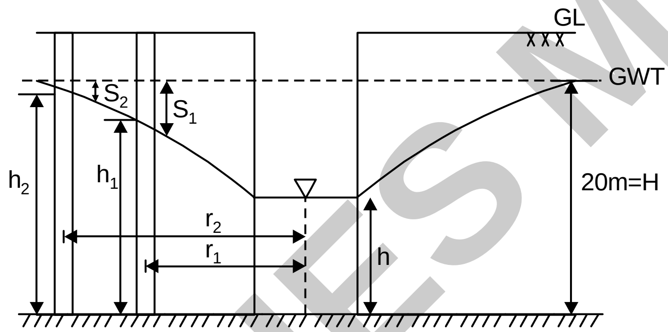
(r_2) distance of second observation well = 100m

(S_1) drawdown in first observation well = 5m

(S_2) drawdown in second observation well = 1m

(H) thickness of unconfined aquifer/initial water table height = 20m

(Q) puming rate/discharge (m^3/day) = ?



(h_1) height of water table at $r_1 = H - S_1$

$= 20 - 5 = 15$ m

(h_2) height of water table at $r_2 = H - S_2$

$= 20 - 1 = 19$m

∴ At steady state we have

$$Q = \frac{\pi k\left(h_2^2 - h_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$Q = \frac{\pi \times 10\left(19^2 - 15^2\right)}{\ln\frac{100}{10}} = 1854.61$$

23. The following table gives various components of Municipal Solid Waste (MSW) and a list of treatment/separation techniques.

Component of MSW

P. Ferrous metals

Q. Aluminum and copper

R. Food waste

S. Cardboard

Treatment/separation technique

i. Incineration

ii. Rapid composting

iii. Eddy current separator

iv. Magnetic separator

The CORRECT match is

(a) P-iv, Q-iii, R-i, S-ii

(b) P-iii, Q-iv, R-i, S-ii

(c) P-iv, Q-iii, R-ii, S-i

(d) P-iii, Q-iv, R-ii, S-i

Sol. (c)

I. Various MSW separation techniques

(a) Screening process

(b) Size reduction process

- Reduction in size of MSW can be done using following mechanical devices:

  (a) Shredders

  (b) Flail mill hammer

  (c) Glass cruster

(c) Density separation process

- It is the process of separation of light materials such as paper and plastic from heavy materials such as metals.
- It is carried out using air density separators.

(d) Magnetic separation process

- It is the process of separating ferrous metals from MSW.

- It is carried out using magnetic separators.

(e) Current separation process

- It is the process of separating aluminium and copper from MSW.
- It is carried out using Eddy current separator.

II. Various MSW treatment technique

(a) Sanitary landfills

(b) Composting

(c) Incineration

(d) Pyrolysis

Hence, correct option is (c).

24. Concrete of characteristic strength 30 MPa is required. If 40 specimens of concrete cubes are to be tested, the minimum number of specimens having at least 30 MPa strength should be

(a) 38 (b) 39

(c) 35 (d) 37

Sol. (a)

Characteristic strength = 30 MPa

No. of cubes = 40

Characteristic strength is the strength below which not more than 5% of the test results are expected to fall.

$\Rightarrow$ 95% of the test results are having strength $\geq$ characteristic strength.

$0.95 \times 40 = 38$

Hence, the correct option is (a).

25. Consider a balanced doubly-reinforced concrete section. If the material and other sectional properties remain unchanged, for which of the following cases will the section becomes under-reinforced?

(a) Area of tension reinforcement is decreased.

(b) Area of compression reinforcement is decreased.

(c) Area of tension reinforcement is increased.

(d) Area of compression reinforcement is increased.

Sol. (a, d)

In a doubly reinforced balance section, the A_{st} and A_{sc} are such that the depth of neutral axis is $x_{u,lim}$ or x_{bal}.

To make the above section an under-reinforced section, the strength of compression side has to be increased compared to that of tension side.

This can be done by either increasing the amount of compression reinforcement or decreasing the amount of tension reinforcement.

Hence, the correct options are (a) and (d).

26. The number of trains and their corresponding speeds for a curved Broad Gauge section with 437 m radius are

- 20 trains travel at a speed of 40 km/hr
- 15 trains travel at a speed of 50 km/hr
- 12 trains travel at a speed of 60 km/hr
- 8 trains travel at a speed of 70 km/hr
- 3 trains travel at a speed of 80 km/hr

If the gauge (center-to-center distance between the rail heads) is taken as 1750 mm, the required equilibrium cant (in mm) will be ________ (rounded off to the nearest integer).

Sol. (88)

$$V_{eq} = \frac{20\times40+15\times50+12\times60+8\times70+3\times80}{20+15+12+8+3}$$

$$= 52.93 \text{ km/hr}$$

$$\therefore \quad \text{Equilibrium cant} = \frac{GV_{eq}^2}{127\,R}$$

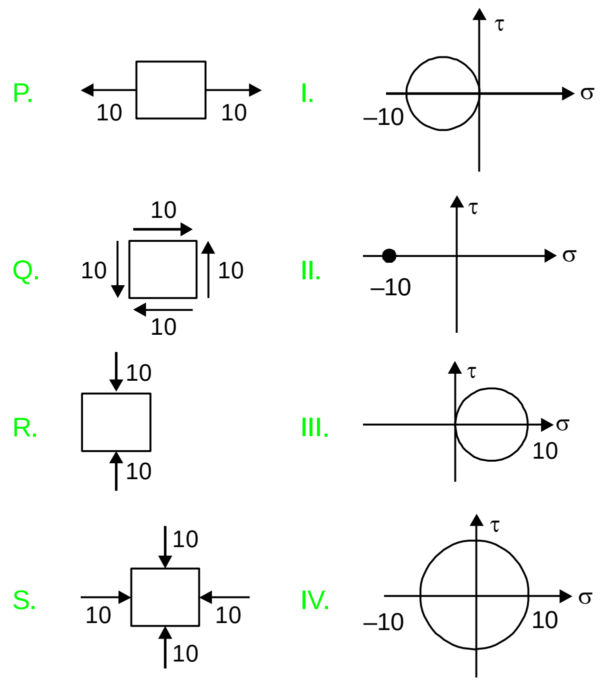
$$= \frac{1.75\times52.93^2}{127\times437}$$

$$= 0.08834 \text{ m}$$

$$\approx 88.3 \text{ mm}$$

$$\text{Closest integer} = 88 \text{ mm} < 165 \text{ mm}$$

27. Find the correct match between the plane stress states and the Mohr's circles.



(a) P-I, Q-IV, R-III, S-II

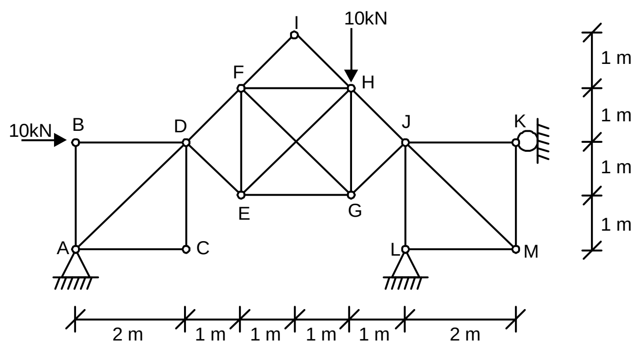
(b) P-III, Q-II, R-I, S-IV

(c) P-I, Q-II, R-III, S-IV

(d) P-III, Q-IV, R-I, S-II

Sol. **(d)**

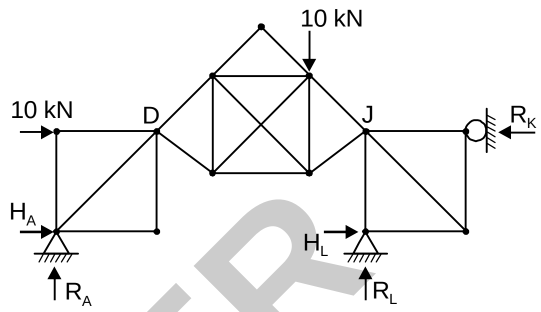
28. The plane truss shown in the figure has 13 joints and 22 members. The truss is made of a homogeneous, prismatic, linearly elastic material. All members have identical axial rigidity. A to M indicate the joints of the truss. The truss has pin supports at joints A and L and roller support at joint K. The truss is subjected to a 10 kN vertically downward force at joint H and a 10 kN horizontal force in the rightward direction at joint B as shown.



(Figure NOT to scale)

The magnitude of the reaction (in kN) at the pin support L is _____ (rounded off to 1 decimal place).

Sol. **(7.5)**



$$\sum F_V = 0 \Rightarrow R_A + R_L - 10 = 0 \quad \text{...(i)}$$

$$\sum F_H = 0 \Rightarrow H_A + H_L - R_K + 10 = 0 \quad \text{...(ii)}$$

$$\sum M_A = 0 \Rightarrow 10 \times 2 + 10 \times 5 - R_L \times 6 - R_K \times 2 = 0 \quad \text{...(iii)}$$

BM at D = 0 $\Rightarrow R_A \times 2 - H_A \times 2 = 0$ ...(iv)

BM at J = 0 $\Rightarrow H_L \times 2 = 0$ ...(v)

From (v), $H_L = 0$

From (iv), $R_A = H_A$

From (ii), $R_A + 0 - R_K + 10 = 0 \Rightarrow R_K = R_A + 10$

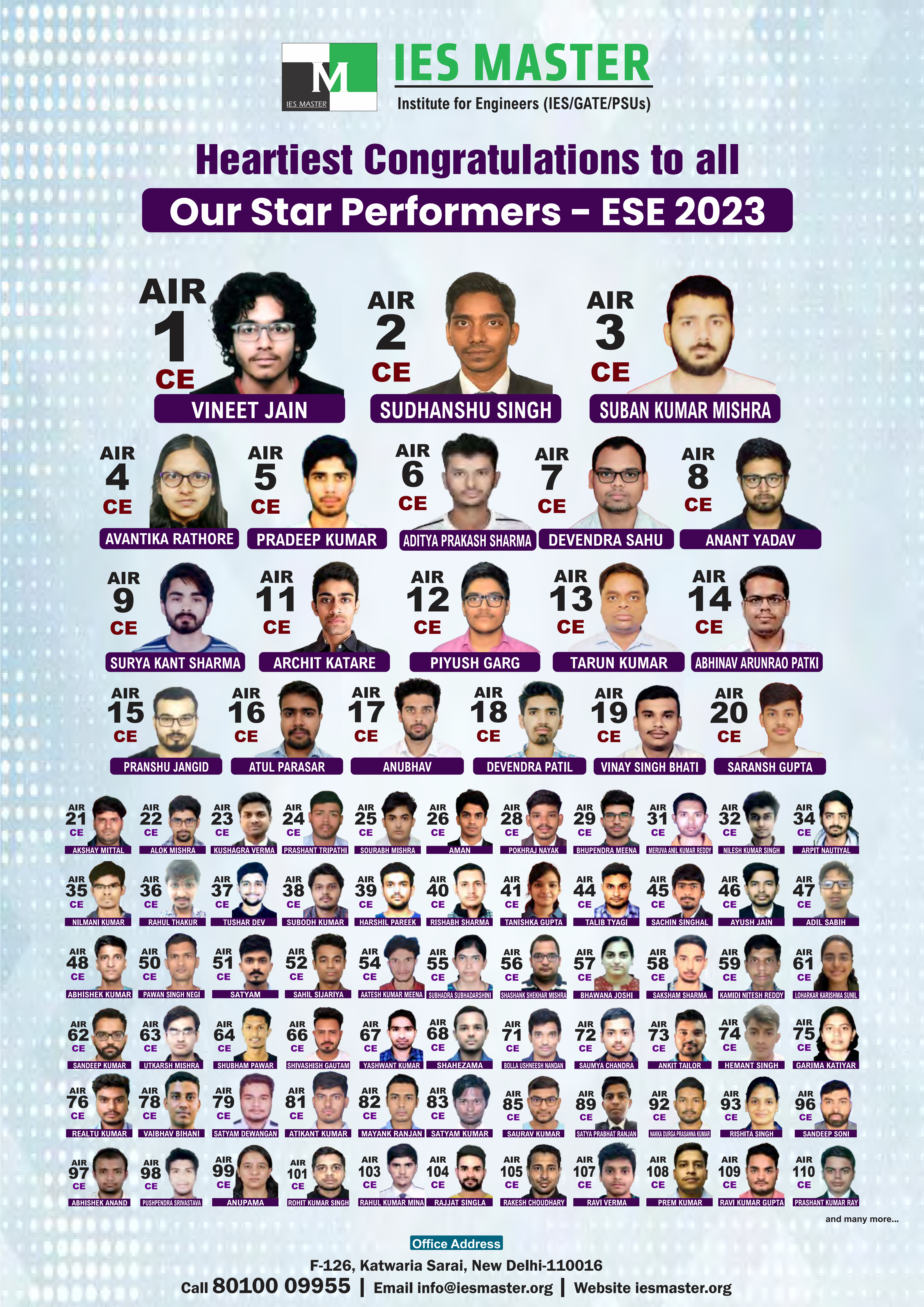
From (iii), $70 - 6R_L - 2(R_A + 10) = 0$

$$\Rightarrow 50 - 6R_L - 2R_A = 0 \quad \text{...}(\alpha)$$

From (i), $2R_A + 2R_L - 20 = 0$ ...(β)

$$30 - 4R_L = 0 \Rightarrow \boxed{R_L = 7.5 \text{ kN}}$$

29. A homogeneous shaft PQR with fixed supports at both ends is subjected to a torsional moment T at point Q, as shown in the figure. The polar moments of inertia of the portions PQ and QR of the shaft with circular cross-sections are J_1 and J_2, respectively. The torsional moment reactions at the supports P and R are T_P and T_R, respectively.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AIR 7 CE DEVENDRA SAHU
AIR 8 CE ANANT YADAV
AIR 9 CE SURYA KANT SHARMA
AIR 11 CE ARCHIT KATARE
AIR 12 CE PIYUSH GARG
AIR 13 CE TARUN KUMAR
AIR 14 CE ABHINAV ARUNRAO PATKI
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AIR 25 CE SOURABH MISHRA
AIR 26 CE AMAN
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AIR 29 CE BHUPENDRA MEENA
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AIR 58 CE SAKSHAM SHARMA
AIR 59 CE KAMIDI NITESH REDDY
AIR 61 CE LOHARKAR KARISHMA SUNIL
AIR 62 CE SANDEEP KUMAR
AIR 63 CE UTKARSH MISHRA
AIR 64 CE SHUBHAM PAWAR
AIR 66 CE SHIVASHISH GAUTAM
AIR 67 CE YASHWANT KUMAR
AIR 68 CE SHAHEZAMA
AIR 71 CE BOLLA USHNEESH NANDAN
AIR 72 CE SAUMYA CHANDRA
AIR 73 CE ANKIT TAILOR
AIR 74 CE HEMANT SINGH
AIR 75 CE GARIMA KATIYAR
AIR 76 CE REALTU KUMAR
AIR 78 CE VAIBHAV BIHANI
AIR 79 CE SATYAM DEWANGAN
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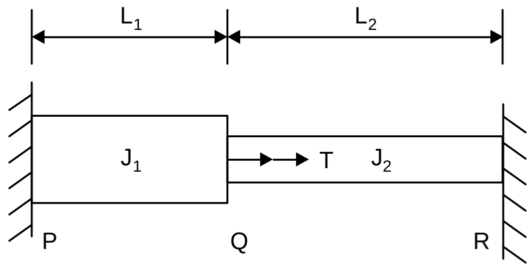


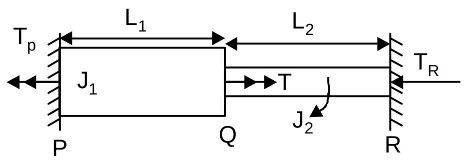
(Figure NOT to scale)

If T_P/T_R = 4 and J_1/J_2 = 2, the ratio of the length L_1/L_2 is

(a) 4.00 (b) 0.25

(c) 0.50 (d) 2.00

Sol. (c)



Given:

$$\frac{T_P}{T_R} = 4 \ \& \ \frac{J_1}{J_2} = 2$$

As P & R are fixed supports, hence

$$|\phi_{QP}| = |\phi_{RQ}|$$

$$\frac{T_P \times L_1}{J_1 \times G_1} = \frac{T_R \times L_2}{G_2 \times J_2}$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{T_R}{T_P} \times \frac{J_1}{J_2} \times \left(\frac{G_1}{G_2}\right)^{1}$$

$$\frac{L_1}{L_2} = \frac{1}{4} \times 2 = \frac{1}{2} = 0.50$$

30. The free mean speed is 60 km/hr on a given road. The average space headway at jam density on this road is 8 m. For a linear speed-density relationship, the maximum flow (in veh/hr/lane) expected on the road is

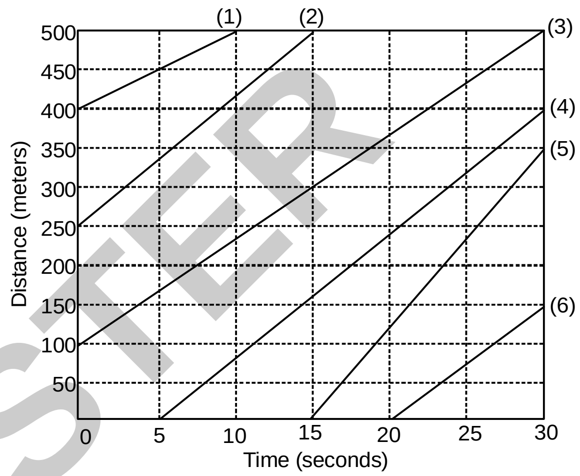
(a) 2075 (b) 938

(c) 1038 (d) 1875

Sol. (d)

$$q_{max} = \frac{V_f K_j}{4} = \frac{60 \times \frac{1000}{8}}{4} = 1875 \text{ veh/hr}$$

31. The figure presents the trajectories of six vehicles within a time-space domain. The number in the parentheses represents unique identification of each vehicle.



(Figure NOT to scale)

The mean speed (in km/hr) of the vehicles in the entire time-space domain is _____ (rounded off to the nearest integer).

Sol. (57)

$$\text{Mean speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{100+250+400+400+350+150}{10+15+30+25+15+10}$$

$$= 15.714 \text{ m/sec}$$

$$= 56.57 \text{ km/h}$$

$$\simeq 57 \text{ km/h}$$

32. The initial cost of an equipment is Rs. 1,00,000. Its salvage value at the end of accounting life of 5 years is Rs. 10,000. The difference in depreciation (in Rs.) computed using 'double-declining balance method' and 'straight line method' of depreciation in Year-2 is _______ (in positive integer).

Sol. (6000)

Initial cost = 100000

Salvage value = 10,000

Accounting life = 5 years

(i) **Depreciation using straight line method**

Annual depreciation

$$= \frac{\text{Initial cost – Salvage value}}{\text{Accounting life or Service life}}$$

$$= \frac{100000-10000}{5} = 18000$$

(ii) **Depreciation using double declining balance method**

In this method asset is assumed to loose its value by a fixed factor of the book value i.e. double the straight line rate.

Fixed factor for double declining balance method = FDDB

$$\text{FDDB} = \frac{2}{n}, \text{ here } n = 5 \text{ years}$$

$$\therefore \quad \text{FDDB} = \frac{2}{5} = 0.4$$

Depreciation for first year = FDDB × Initial cost

= 0.4 × 100000

= 40000

Book value at end of 1st year = Initial cost – Depreciation

= 100000 – 40000

= 60000

Depreciation for 2nd year = 60000 × 0.4 = 24000

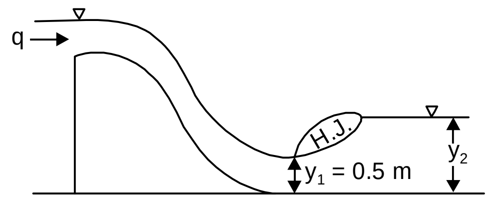
Difference in depreciation between double declining balance and straight line method

= 24000 – 18000

= 6000

33. A spillway has unit discharge of 7.5 m^3/s/m. The flow depth at the downstream horizontal apron is 0.5 m. The tail water depth (in meters) required to form a hydraulic jump is _____ (rounded off to 2 decimal places).

Sol. **(4.55)**



Discharge = 7.5 m^3/sec/m

Flow depth at downstream horizontal apron (y_1) = 0.5 m.

$$v_1 = \frac{q}{y_1} = \frac{7.5}{0.5} = 15 \text{ m/s}$$

Initial Froude number (Fr_1)

$$= \frac{v_1}{\sqrt{gy_1}} = \frac{15}{\sqrt{9.81\times 0.5}}$$

$$= 6.7729 > 1$$

Let y_2 be the sequent depth corresponding to initial depth y_1 = 0.5 m

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2}-1\right]$$

or $$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8\times 6.7729^2}-1\right]$$

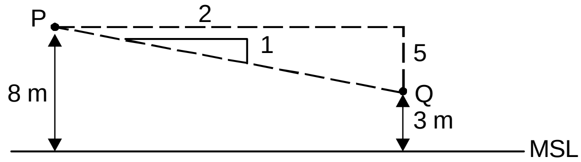
or $$\frac{y_2}{y_1} = 9.0914$$

$$\therefore \quad y_2 = 0.5 \times 9.0914 = 4.5457$$

So, tail water depth after rounding off 2 decimal places = 4.55 m.

34. A bird is resting on a point P at a height of 8 m above the Mean Sea Level (MSL). Upon hearing a loud noise, the bird flies parallel to the ground surface and reaches a point Q which is located at a height of 3 m above MSL. The ground surface has a falling gradient of 1 in 2. Ignoring the effects of curvature and refraction, the horizontal distance (in meters) between points P and Q is _____ (in integer).

Sol. **(10)**



Bird travelled parallel to ground and slope of ground is given as 1 in 2.

$\therefore$ Horizontal distance $\Rightarrow$ 10 m.

35. The ordinates of a 1-hour unit hydrograph (UH) are given below.

Time (hours)	Ordinates of 1-hour UH (m^3/s)
0	0
1	13
2	50
3	80
4	95
5	85
6	55
7	35
8	15
9	10
10	3
11	0

These ordinates are used to derive a 3-hour UH. The peak discharge (in m^3/s) for the derived 3-hour UH is _______ (rounded off to the nearest integer).

Sol. (87)

t	I 1h UH	II 1h UH lagged by 1 hr	III 1h UH lagged by 2 hr	3h UH = $\frac{I + II + III}{3}$
0	0			0
1	13	0		13/3 = 4.33
2	50	13	0	63/3 = 21
3	80	50	13	47.6
4	95	80	50	75
5	85	95	80	86.67
6	55	85	95	⋮
7	35	55	85	⋮
8	15	⋮	⋮	⋮
9	10	⋮	⋮	⋮
10	3	⋮	⋮	⋮
11	0	⋮	⋮	⋮

36. A flow velocity field $\vec{V} : \vec{V}(x, y)$ for a fluid is represented by

$$\vec{V} = 3\hat{i} + (5x)\,\hat{j}$$

In the context of the fluid and flow, which one of the following statements is CORRECT?

(a) The fluid is incompressible and the flow is rotational.

(b) The fluid is incompressible and the flow is irrotational.

(c) The fluid is compressible and the flow is irrotational.

(d) The fluid is compressible and the flow is rotational.

Sol. (a)

$$\vec{v} = 3\hat{i} + (5x)\,\hat{j}$$

Here, u = 3, v = (5x)

Check for incompressibility

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial(3)}{\partial x} + \frac{\partial(5x)}{\partial y} = 0$$

Since, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Fluid is incompressible.

Check for rotationality

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= \frac{1}{2}\left[\frac{\partial(5x)}{\partial x} - \frac{\partial(3)}{\partial y}\right]$$

$$= \frac{1}{2}[5 - 0] = 2.5 \neq 0$$

So, flow is rotational.

37. A vector field $\vec{p}$ and a scalar field r are given by

$$\vec{p} = (2x^2 - 3xy + z^2)\hat{i} + (2y^2 - 3yz + x^2)\hat{j} + (2z^2 - 3xz + x^2)\,\hat{k}$$

$r = 6x^2 + 4y^2 - z^2 - 9xyz - 2xy + 3xz - yz$

Consider the statements P and Q.

P: Curl of the gradient of the scalar field r is a null vector.

Q: Divergence of curl of the vector field $\vec{p}$ is zero.

Which one of the following options is CORRECT?

(a) Both P and Q are FALSE

(b) Both P and Q are TRUE

(c) P is TRUE and Q is FALSE

(d) P is FALSE and Q is TRUE

Sol. (b)

Curl of gradient r

$= \text{Curl}(\nabla r)$

$= \nabla \times \nabla r$

= 0 ($\because$ curl of a gradient vector is the zero vector)

Divergence of curl of the vector $\vec{p}$

$= \nabla \cdot (\nabla \times \vec{p})$

= 0 ($\because$ The divergence of the curl is zero in all circumstances)

38. For assessing the compliance with the emissions standards of incineration plants, a correction needs to be applied to the measured concentrations of air pollutants. The emission standard (based on 11% Oxygen) for HCl is 50 mg/Nm^3 and the measured concentrations of HCl and Oxygen in flue gas are 42 mg/Nm^3 and 13%, respectively.

Assuming 21% oxygen in air, the CORRECT statement is:

(a) No compliance, as the Oxygen is greater than 11% in the flue gas.

(b) Compliance is there, as the corrected HCl emission is lesser than the emission standard.

(c) Compliance is there, as there is no need to apply the correction since Oxygen is greater than 11% and HCl emission is lesser than the emission standard.

(d) No compliance, as the corrected HCl emission is greater than the emission standard.

Sol. (d)

Given: Emission standard (based on 11% oxygen)

For HCl = 50 mg/Nm^3

Measured concentration of HCl and oxygen in flue gas are 42 mg/Nm^3 and 13% respectively.

As per CPCB,

Calculated emission concentration at the standard percentage oxygen concentration (ES)

$$ES = \frac{21-OS}{21-OM} \times EM$$

EM = Measured emission concentration at the standard percentage oxygen concentration

OS = Standard oxygen concentration

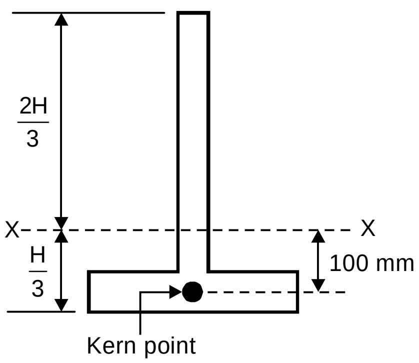
OM = Measured oxygen concentration

So,

$$ES = \frac{21-11}{21-13} \times 42 = 52.5 \text{ mg/Nm}^3 > 50 \text{ mg/Nm}^3$$

Here, no compliance as the corrected HCl emission is greater than the emission standard.

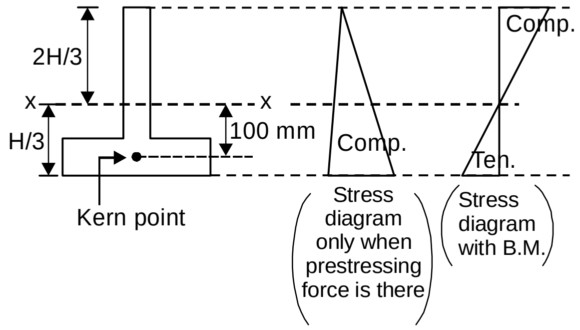
39. An inverted T-shaped concrete beam (B1) in the figure, with centroidal axis X–X, is subjected to an effective prestressing force of 1000 kN acting at the bottom kern point of the beam cross-section. Also consider an identical concrete beam (B2) with the same grade of concrete but without any prestressing force.



(Figure NOT to scale)

The additional cracking moment (in kN.m) that can be carried by beam B1 in comparison to beam B2 is __________ (rounded off to the nearest integer).

Sol. (300)



Resultant stress at the bottom fibre in beam B,

$$= \frac{P}{A} + \frac{Pe}{Z_b} - \frac{M_1}{Z_b}$$

$$= \frac{1000 \times 10^3}{A} + \frac{1000 \times 10^3}{Z_b} - \frac{M_1}{Z_b}$$

For no crack, stress at bottom = f_{cr}

$$\Rightarrow \frac{1000 \times 10^3}{A} + \frac{1000 \times 10^3 \times 100}{Z_b} - \frac{M_1}{Z_b} = -f_{cr} \quad \ldots(i)$$

Resultant stress at the bottom fibre in beam B_2 i.e.

when no prestressing force is here = $-\frac{M_2}{Z_b}$

For no crack, stress at bottom = f_{cr}

$$\Rightarrow -\frac{M_2}{Z_b} = -f_{cr} \quad \ldots(ii)$$

Also, for beam B_1, without the moment M_1

$$\frac{P}{A} - \frac{Pe}{Z_t} = 0$$

$$\Rightarrow \frac{I}{A} - \frac{e}{Z_t} = 0$$

$$\Rightarrow \frac{I}{A} = \frac{e}{Z_t}$$

$$\Rightarrow A = \frac{Z_t}{e}$$

$$\frac{Z_b}{Z_t} = \frac{(I/y_b)}{(I/y_t)} = \frac{y_t}{y_b} = \frac{2H}{3(H/3)} = 2$$

$$\Rightarrow Z_t = 0.5Z_b$$

$$\Rightarrow A = \frac{0.5Z_b}{e}$$

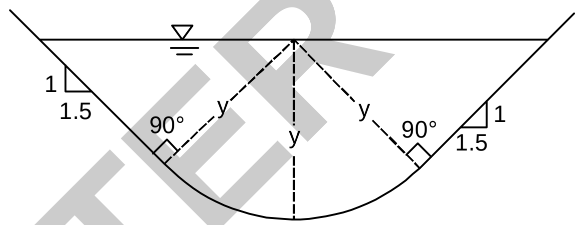
Adding equation (i) and (ii)

$$\Rightarrow \frac{1000 \times 10^3}{A} + \frac{1000 \times 10^3 \times 100}{Z_b} - \frac{M_1}{Z_b} + \frac{M_2}{Z_b} = 0$$

$$\Rightarrow \frac{1000 \times 10^3 \times 100}{0.5Z_b} + \frac{1000 \times 10^3}{Z_b} - \frac{(M_1 - M_2)}{Z_b} = 0$$

$$\Rightarrow M_1 - M_2 = 300 \text{ kN.m}$$

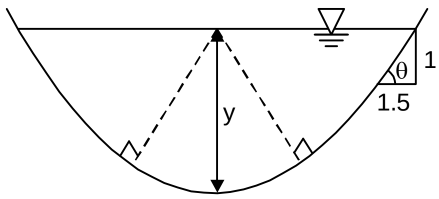
40. A standard round bottom triangular canal section as shown in the figure has a bed slope of 1 in 200. Consider the Chezy's coefficient as 150 $m^{1/2}/s$.



(Figure NOT to scale)

The normal depth of flow, y (in meters) for carrying a discharge of 20 m^3/s is _____ (rounded off to 2 decimal places).

Sol. (1.10)



$$\cot\theta = 1.5$$

$$\theta = 0.588 \text{ rad}$$

$$\text{Area (A)} = y^2(\theta + \cot\theta)$$

$$\Rightarrow A = 2.088y^2$$

$$p = 2y(\theta + \cot\theta)$$

$$\Rightarrow R = \frac{y}{2}$$

Using chezy's equation :

$$Q = AV = A \times C\sqrt{RS}$$

$$\Rightarrow 20 = 2.088y^2 \times 150\sqrt{\frac{y}{2} \times \frac{1}{200}}$$

$$\Rightarrow y^{5/2} = \frac{20 \times 2 \times 10}{2.088 \times 150}$$

$$\Rightarrow y = 1.102\text{m}$$

41. The total primary consolidation settlement (S_c) of a building constructed on a 10 m thick saturated clay layer is estimated to be 50 mm. After 300 days of the construction of the building, primary consolidation settlement was reported as 10 mm. The additional time (in days) required to achieve 50% of S_c will be _____ (rounded off to the nearest integer).

Sol. **(1575)**

(S_c) Ultimate primary consolidation settlement = 50mm

(S_i) Consolidation settlement after 300 day = 10mm

(U) degree of consolidation at 10mm settlement

$$U_1 = \frac{S_i \times 100}{S_c} = \frac{10}{50} \times 100 = 20\%$$

(t_1) time to achieve 10mm settlement = 300 days

Given, (U_2) degree of consolidation = 50%

$$\therefore \quad T_v = \frac{C_v t}{d^2}$$

$$T_v \propto t$$

where, (T_v) time factor and (t) time needed to achieve a particular degree of consolidation

$$\frac{T_{v1}}{T_{v2}} = \frac{t_1}{t_2}$$

$$\therefore \quad T_v = \frac{\pi}{4}(U^2) \quad \text{for} \quad U \le 60\%$$

$$\text{So,} \quad \frac{(U_1)^2}{(U_2)^2} = \frac{t_1}{t_2},$$

$$\frac{(20)^2}{(50)^2} = \frac{300}{t_2}$$

(t_2) time needed to achieve 50% degree of consolidation = 1875 days

So, additional time needed = 1875 – 300

$$\boxed{= 1575 \text{ days}}$$

42. What are the eigenvalues of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$?

(a) –5, –1, 2
(b) –5, 1, 2
(c) 1, 3, 4
(d) 1, 2, 5

Sol. **(d)**

Sum of eigen values

= Trace (A)

= 2 + 4 + 2 = 8

Product of eigen values

= det (A)

= 2(8 – 1) – (2 – 1) + 1(1 – 4)

= 10

Hence option (d) is correct.

43. Activated carbon is used to remove a pollutant from wastewater in a mixed batch reactor, which follows first-order reaction kinetics.

At a reaction rate of 0.38/day, the time (in days) required to remove the pollutant by 95% is _____ (rounded off to 1 decimal place).

Sol. **(7.9)**

Let,

N_o = Number of pollutant present in waste water intially i.e. at t = 0

N_t = Number of pollutant present in waste water at time, 't'

Treatment of activated carbon to remove pollutant in mixed batch reactor follows 1st order kinetics.

i.e. $\boxed{N_t = N_o e^{-kt}}$ ...(i)

As, removal efficiency at any time 't' = 95%

$$\text{i.e.} \quad \frac{N_o - N_t}{N_o} \times 100 = 95$$

$$\text{So,} \quad \boxed{\frac{N_t}{N_o} = 0.05}$$

On putting in eq. (i)

So, 0.05 = $e^{-0.38t}$

On solving,

$$\boxed{t = 7.88 \text{ days} \approx 7.9 \text{ days}}$$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44. A water treatment plant treats 25 MLD water with a natural alkalinity of 4.0 mg/L (as $CaCO_3$). It is estimated that, during coagulation of this water, 450 kg/day of calcium bicarbonate ($Ca(HCO_3)_2$) is required based on the alum dosage.

Consider the atomic weights as: Ca-40, H-1, C-12, O-16.

The quantity of pure quick lime, CaO (in kg) required for this process per day is _____ (rounded off to 2 decimal places).

Sol. (99.54)

Natural alkalinity = 4 mg/l as $CaCO_3$

$$\text{Alkalinity req} = \left(\frac{450\times10^3 \text{ g/day}}{25\times10^6 \, l/d \times \text{eq wt of } Ca(HCO_3)_2}\times \text{eq wt of } CaCO_3\right)\times 10^3 \frac{mg}{l} \text{ as } CaCO_3$$

$$= \frac{450\times10^3\times10^3}{25\times10^6\times81}\times50$$

$$= \frac{450\times50}{25\times81} = 11.11 \, mg/l \text{ as } CaCO_3$$

$\Rightarrow$ Alkalinity needed to be provided through outside source = (11.11 – 4) = 7.11 mg/l as $CaCO_3$

$$= \frac{7.11}{50} \text{ milieq}/l$$

$$\Rightarrow \text{CaO required} = \frac{7.11}{50} \text{ milieq}/l$$

$$= \left[\frac{7.11}{50}\times \text{eq.wt of CaO}\right]\frac{mg}{l}$$

$$= \left[\frac{7.11}{50}\times 28\right]\frac{mg}{l}\times 25\times10^6 \, l/d$$

= 99.54 kg/d

45. The following data is obtained from an axle load survey at a site:

Average rear axle load = 12000 kg

Number of commercial vehicles = 800 per day

The pavement at this site would be reconstructed over a period of 5 years from the date of survey. The design life of the reconstructed pavement is 15 years. Use the standard axle load as 8160 kg and the annual average vehicle growth rate as 4.0%. Assume that Equivalent Wheel Load Factor (EWLF) and Vehicle Damage Factor (VDF) are equal.

The cumulative standard axle (in msa) for the pavement design is ______ (rounded off to 2 decimal places).

Sol. (33.27)

$$\text{VDF} = \text{EWLF} = \left(\frac{\text{Axle load}}{\text{Standard axle load}}\right)^4$$

$$= \left(\frac{12000}{8160}\right)^4 = 4.677$$

$$A = P(1+r)^x$$

$$= 800(1.04)^5 = 973.322$$

$$\text{CSA} = \frac{365A((1+r)^n - 1)}{r}\text{LDF}\times\text{VDF}$$

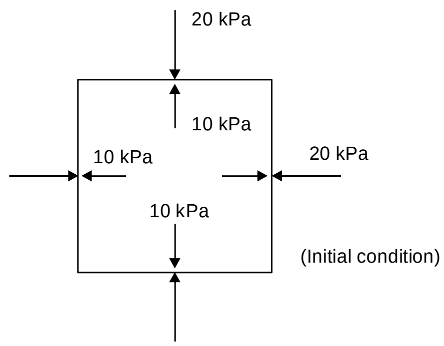
$$= \frac{365\times973.322[1.04^{15}-1]}{0.04}\times1\times4.677$$

= 33.27 msa

Assuming LDF to be 1.

46. A soil sample was consolidated at a cell pressure of 20 kPa and a back pressure of 10 kPa for 24 hours during a consolidated undrained (CU) triaxial test. The cell pressure was increased to 30 kPa on the next day and it resulted in the development of pore water pressure of 1 kPa. The soil sample failed when the axial stress was gradually increased to 50 kPa. The pore water pressure at failure was recorded as 21 kPa. The value of Skempton's pore pressure parameter B for the soil sample is _______ (rounded off to 2 decimal places).

Sol. (0.1)



When the cell pressure was increased to 30 kPa

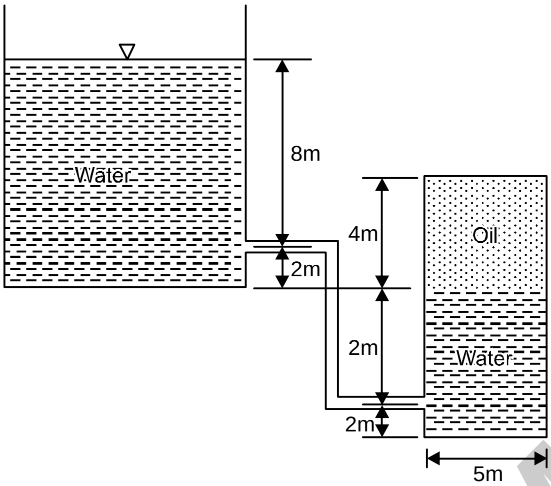
$$\Delta\sigma_3 = (30-20) = 10 \text{ kPa}$$

$$\Delta u_1 = 1 \text{ kPa}$$

$$\Rightarrow \quad B = \frac{\Delta u_1}{\Delta \sigma_3} = \frac{1}{10} = 0.1$$

$$\boxed{B = 0.1}$$

47. A 5 m × 5 m closed tank of 10 m height contains water and oil and is connected to an overhead water reservoir as shown in the figure. Use γ_w = 10 kN/m^3 and specific gravity of oil = 0.8.

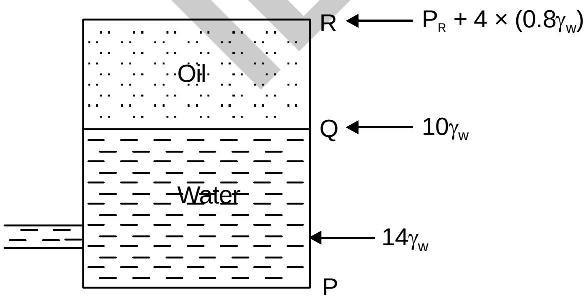


(Figure NOT to scale)

The total force (in kN) due to pressure on the side PQR of the tank is _____ (rounded off to the nearest integer).

Sol. (5580)

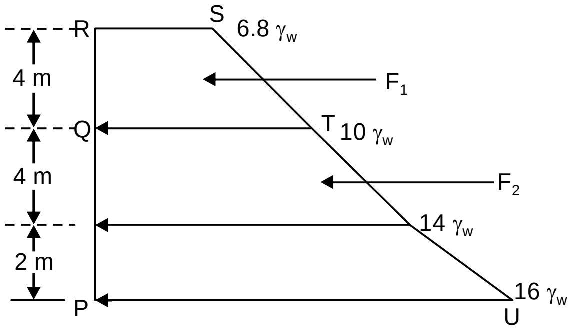
Considering PQR



$$P_R + 4 \times (0.8\gamma_w) = 10\,\gamma_w$$

$$\Rightarrow P_R = (10 - 3.2)\gamma_w = 6.8\,\gamma_w$$

Now,



Considering force prism QRST

$$F_1 = \frac{(6.8 + 10)}{2}\gamma_w \times 4 \times 5 = 168\,\gamma_w = 1680\text{ kN}$$

Considering force prism PQTU

$$F_2 = \frac{(10 + 16)}{2}\gamma_w \times 6 \times 5 = 3900\text{ kN}$$

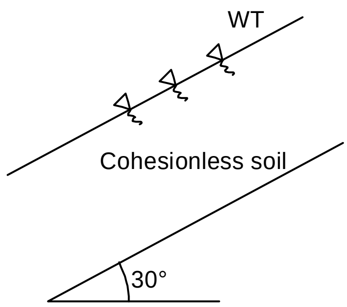
So, $F = F_1 + F_2$ = 1680 + 3900 = 5580 kN

48. A infinite slope is made up of cohesionless soil with seepage parallel to and up to the sloping surface. The angle of slope is 30° with respect to horizontal ground surface. The unit weights of the saturated soil and water are 20 kN/m^3 and 10 kN/m^3, respectively.

The minimum angle of shearing resistance of the soil (in degrees) for the critically stable condition of the slope is ______ (rounded off to the nearest integer).

Sol. (49)

$\therefore$ Seepage is occuring upto the ground surface



Given, (γ_{sat}) saturated unit weight = 20 kN/m^3

(γ_w) unit weight of water = 10 kN/m^3

(β) slope angle = 30°

(ϕ) angle of shearing resistance of soil = ?

∴ Water table is at ground surface & seepage is occuring so,

$$\text{(FOS) factor of safety} = \frac{\gamma_{sub}}{\gamma_{sat}}\frac{\tan\phi}{\tan\beta}$$

$$\text{For critical condition, FOS} = 1 = \left(\frac{\gamma_{sat}-\gamma_w}{\gamma_{sat}}\right)\times\frac{\tan\phi}{\tan\beta}$$

$$1 = \left(\frac{20-10}{20}\right)\times\frac{\tan\phi}{\tan 30°}$$

$$\boxed{\text{Angle of shearing resistance } (\phi) = 49.106° \cong 49°}$$

49. A 2 m × 2 m tank of 3 m height has inflow, outflow and stirring mechanisms. Initially, the tank was half-filled with fresh water. At t = 0, an inflow of a salt solution of concentration 5 g/m^3 at the rate of 2 litre/s and an outflow of the well stirred mixture at the rate of 1 litre/s are initiated. This process can be modelled using the following differential equation:

$$\frac{dm}{dt}+\frac{m}{6000+t} = 0.01$$

where, m is the mass (grams) of the salt at time t (seconds). The mass of the salt (in grams) in the tank at 75% of its capacity is ______ (rounded off to 2 decimal places).

Sol. (25)

We know that $\frac{dx}{dt}+Px = Q$

$$\Rightarrow \quad \frac{dm}{dt}+Pm = Q$$

$$P = \frac{1}{6000+t}$$

$$Q = 0.01$$

$$\text{Integration factor (I.F.)} = e^{\int P\,dt} = e^{\int \frac{1}{6000+t}dt}$$

$$= e^{\ln(6000+t)} = 6000+t$$

$$\Rightarrow \quad m.(\text{I.F.}) = \int Q.(\text{I.F.})dt + C$$

$$m(6000+t) = \int 0.01(6000+t)\,dt + C$$

$$m(6000+t) = 0.01\left[6000t+\frac{t^2}{2}\right]+C$$

at t = 0, m = 0

[Since there is Fresh water in tank initially]

$$\Rightarrow \quad C = 0$$

$$\Rightarrow \quad m = \frac{0.01\left(6000t+\frac{t^2}{2}\right)}{(6000+t)}$$

From volume balance

Accumulation = Inflow – Ourtflow

$$2\times2\times(3\times0.25)m^3 = (2-1)\frac{l}{5}\times t_{sec}\times10^{-3}\frac{m^3}{l}$$

$$\Rightarrow \quad t = 3000 \text{ sec.}$$

Hence, at 3000 sec. i.e. when tank is 75% full.

$$m = \frac{0.01\left(6000\times3000+\frac{(3000)^2}{2}\right)}{6000+3000}$$

$$m = 0.01[2000+500]$$

$$m = 25 \text{ gram}$$

50. The return period of a large earthquake for a given region is 200 years. Assuming that earthquake occurrence follows Poisson's distribution, the probability that it will be exceeded at least once in 50 years is ________ %. (rounded off to the nearest integer).

Sol. (22)

$$T = 200 \text{ yrs}$$

$$\lambda = \frac{1}{200}$$

Probability of non occurrence in 50 years

$$= e^{-\lambda t}$$
$$= 0.7788$$

Probability of occurrence atleast once

$$= 1 - 0.7788$$
$$= 0.2212$$
$$\simeq 22.12\%$$

51. Which of the following statement(s) is/are CORRECT?

(a) Swell potential of soil decreases with an increase in the shrinkage limit.

(b) In electrical resistivity tomography, the depth of current penetration is half of the spacing between the electrodes.

(c) Both loose and dense sands with different initial void ratios can attain similar void ratio at large strain during shearing.

(d) Among the several corrections to be applied to the SPT-N value, the dilatancy correction is applied before all other corrections.

Sol. (a, c)

- Shrinkage limit is defined as the maximum water content at which further reduction in water content of soil does not lead to reduction in the volume of soil as water is being just replaced by air.
- Higher is the shrinkage limit lower is the compressibility (volume change).
- Dilatancy correction is applied to the already corrected N-values for overburden pressure.
- At large value of shearing strain, both the initially loose and initially dense sands approach a constant value of void ratio called critical void ratio.

52. A slab panel with an effective depth of 250 mm is reinforced with 0.2% main reinforcement using 8 mm diameter steel bars. The uniform center-to-center spacing (in mm) at which the 8 mm diameter bars are placed in the slab panel is _________ (rounded off to the nearest integer).

Sol. (100)

Area of reinforcement considering the width of slab as 1 m.

$$A_{st} = \frac{0.2}{100}bd = \frac{0.2}{100}\times 1000\times 250 = 500\text{ mm}^2$$

$$\text{Spacing} = \frac{1000}{\left(\dfrac{A_{st}}{\frac{\pi}{4}\times 8^2}\right)} = 100.53\text{ mm}$$

Rounded upto the nearest integer considering safety is 100 mm. Hence, the correct answer is 100.

53. A map is prepared with a scale of 1 : 1000 and a contour interval of 1 m. If the distance between two adjacent contours on the map is 10 mm, the slope of the ground between the adjacent contours is

(a) 35% (b) 30%

(c) 10% (d) 40%

Sol. (c)

Scale $\Rightarrow \frac{1}{1000}$, horizontal distance on map = 10 mm

$\therefore$ Horizontal distance between two contour on ground

$$= \frac{10}{\text{Scale}} = 10\times\frac{1000}{1}\text{ mm} = 10\text{ m}$$

$$\therefore \text{ Slope} = \frac{\text{Vertical}}{\text{Horizontal}}\times 100 = \frac{1\text{m}}{10\text{ m}}\times 100 = 10\%$$

54. A vertical smooth rigid retaining wall is supporting horizontal ground with dry cohesionless backfill having a friction angle of 30°. The inclinations of failure planes with respect to the major principal plane for Rankine's active and passive earth pressure conditions, respectively, are

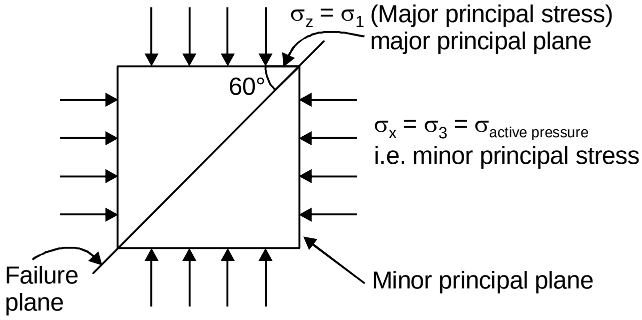
(a) 30° and 30° (b) 60° and 60°

(c) 60° and 30° (d) 30° and 60°

Sol. (b)

Given, (ϕ) friction angle = 30°

For active state

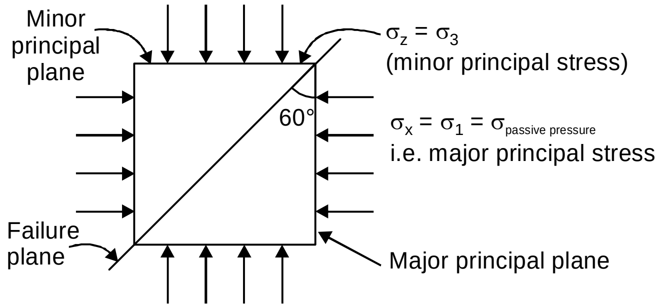


Failure plane inclination with major principal plane (horizontal)

$$\Rightarrow \theta_c = 45° + \frac{\phi}{2} = 45° + \frac{30°}{2}$$

$$\boxed{\theta_c = 60°}$$

For passive state



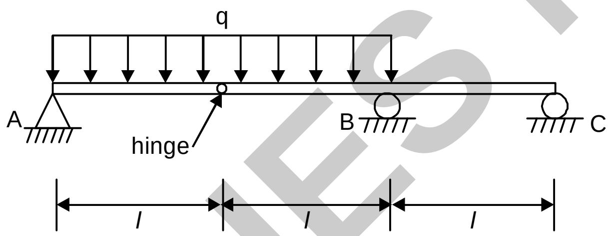
Failure plane inclination with major principal plane

$$(\theta_c) = 45° + \frac{\phi}{2}$$

$$\boxed{\theta_c = 45° + \frac{30°}{2} = 60°}$$

Hence inclination of failure planes with major principal plane for both active & passive conditions is 60°

55. The beam shown in the figure is subjected to a uniformly distributed downward load of intensity q between supports A and B.



Considering the upward reactions as positive, the support reactions are

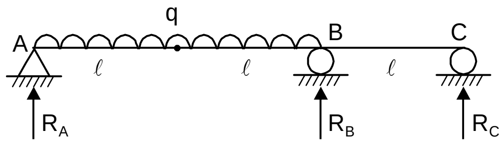
(a) $R_A = -q\ell; R_B = \frac{5q\ell}{2}; R_C = \frac{q\ell}{2}$

(b) $R_A = \frac{q\ell}{2}; R_B = q\ell; R_C = \frac{q\ell}{2}$

(c) $R_A = \frac{q\ell}{2}; R_B = \frac{5q\ell}{2}; R_C = -q\ell$

(d) $R_A = -\frac{q\ell}{2}; R_B = \frac{5q\ell}{2}; R_C = 0$

Sol. (c)



BM at hinge = 0

$$\Rightarrow R_A\ell - \frac{q\ell^2}{2} = 0$$

$$\boxed{R_A = \frac{q\ell}{2}} \text{ upward i.e. (+)ve}$$

$$\sum F_V = 0 \Rightarrow R_A + R_B + R_C = 2q\ell$$

$$\Rightarrow R_B + R_C = 2q\ell - R_A$$

$$= 2q\ell - \frac{q\ell}{2} = 1.5\, q\ell \qquad \text{...(A)}$$

$$\sum M_B = 0$$

$$\Rightarrow R_A(2\ell) - q(2\ell)\ell - R_C\ell = 0$$

$$\frac{q\ell}{2}(2\ell) - 2q\ell^2 - R_C\ell = 0$$

$$-q\ell^2 - R_C\ell = 0$$

$$\boxed{R_C = -q\ell}$$

$$\Rightarrow \boxed{R_C = q\ell} \text{ downward}$$

$\Rightarrow$ From (A), $R_B = 1.5q\ell + q\ell = 2.5\,q\ell$

$$\Rightarrow \boxed{R_B = 2.5\,q\ell} \text{ upward i.e. (+)ve}$$

Hence, correct answer is (c).