IES MASTER
Institute for Engineers (IES/GATE/PSUs)

# GATE 2024

## MECHANICAL ENGINEERING

## Detailed Solution

EXAM DATE: 03-02-2024
AFTERNOON SESSION (02:30 PM-05:30 PM)

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## SECTION : GENERAL APTITUDE

**1.** How many combinations of non-null sets A, B, C are possible from the subsets of {2, 3, 5} satisfying the conditions : (i) A is a subset of B, and (ii) B is a subset of C ?

(a) 28 (b) 27

(c) 19 (d) 18

**Sol–1: (c) MTA**

**2.** Take two long dice (rectangular parallelepiped), each having four rectangular faces labelled as 2, 3, 5 and 7. If thrown, the long dice cannot land on the square faces and has 1/4 probability of landing on any of the four rectangular faces. The label on the top face of the dice is the score of the throw.

If thrown together, what is the probability of getting the sum of the two long dice scores greater than 11 ?

(a) 3/8 (b) 1/16

(c) 1/8 (d) 3/16

**Sol–2: (d)**

The combination of numbers on two long dice to get sum greater than 11.

(5, 7), (7, 5) and (7, 7)

So, the probability

$$= \frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}$$

$$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{3}{16}$$

**3.** In the given text, the blanks are numbered (i)-(iv). Select the best match for all the blanks.

Prof. P ____(i)____ merely a man who narrated stories __(ii)__ in his blackest moments he was capable of self-depreciating humor.

Prof Q __(iii)__ a man who hardly narrated funny stories. __(iv)__ in his blackest moments was he able to find humor.

(a) (i) was (ii) Even (iii) wasn't (iv) Only

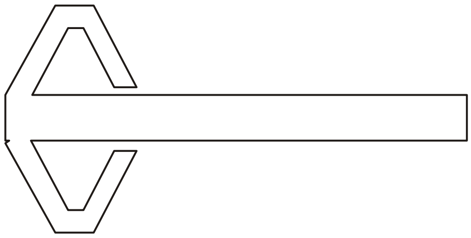
(b) (i) was (ii) Only (iii) wasn't (iv) Even

(c) (i) wasn't (ii) Even (iii) was (iv) Only

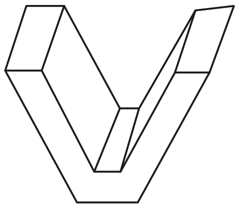
(d) (i) wasn't (ii) Only (iii) was (iv) Even

**Sol–3: (c)**

**4.** A planar rectangular paper has two V-shaped pieces attached as shown below.



This piece of paper is folded to make the following closed three-dimensional object.



The number of folds required to form the above object is

(a) 7 (b) 11

(c) 9 (d) 8

**Sol–4: (c)**

**5.** The real variable x, y, z and the real constants p, q, r satisfy.

$$\frac{x}{pq-r^2}=\frac{y}{qr-p^2}=\frac{z}{rp-q^2}$$

Given that the denominators are non-zero, the value of px + qy + rz is

(a) pqr (b) 0

(c) 1 (d) $p^2+q^2+r^2$

**Sol–5: (b)**

Let $\frac{x}{pq-r^2}=\frac{y}{qr-p^2}=\frac{2}{rp-q^2}=K$

$\Rightarrow$ $x = K(pq - r^2)$

$y = K(qr - p^2)$

$z = K(rp - q^2)$

So, px + qy + rz =

$$K \times p(pq - r^2) + K \times q(qr - p^2) + K \times r(rp - q^2)$$

$$= K(p^2q - pr^2 + q^2r - qp^2 + r^2p - rq^2)$$

$$= K \times 0 = 0$$

**6.** Find the odd one out in the set : {19, 37, 21, 17, 23, 29, 31, 11}.

(a) 29 (b) 21

(c) 37 (d) 23

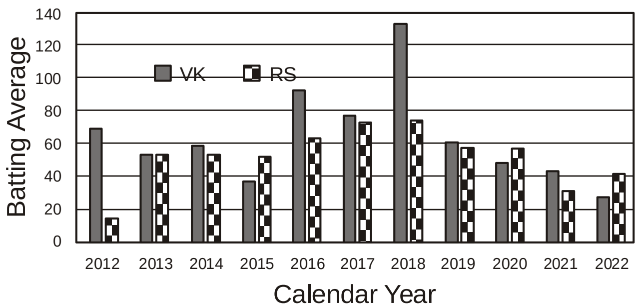
**Sol–6: (b)**

{19, 37, **21**, 17, 23, 29, 31, 11}

This is the set of prime number.

Here '21' is odd one. 21 is non prime number.

**7.** The bar chart gives the batting averages of VK and RS for 11 calendar years from 2012 to 2022. Considering that 2015 and 2019 are world cup years, which one of the following options is true ?




(a) VK's yearly batting average is consistently higher than that of RS between the two world cup years.

(b) VK has a higher yearly batting average than that of RS in every world cup year.

(c) RS has a higher yearly batting average than that of VK in every world cup year.

(d) RS's yearly batting average is consistently higher than that of VK in the last three years.

**Sol–7: (a)**

From graph

In between 2015 to 2019, VK batting average

$$= 35 + 90 + 75 + 130 + 60 = \frac{390}{5} = 78$$

In between 2015 to 2019, RS batting average

$$= 50 + 65 + 70 + 70 + 55 = \frac{320}{5} = 64$$

So, VK yearly batting average consistantly higher then RS between two world cup years (2015 and 2019).

**8.** In the following series, identify the number that needs to be changed to form the Fibonacci series.

1, 1, 2, 3, 6, 8, 13, 21, ..

(a) 8 (b) 21

(c) 13 (d) 6

**Sol–8: (d)**

The Fibonacci series is the series of numbers

1, 1, 2, 3, 5,8, 13, 21, ....

The next number is determined by adding the two numbers before it.

1 + 1 = 2

2 + 1 = 3

2 + 3 = $\boxed{5}$

3 + 5 = 8

5 + 8 = 13

8 + 13 = 21

So, number **'6'** needs to be changed to form the Fibonacci series.

**9.** If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [smile $\rightarrow$ giggle $\rightarrow$ laugh] is analogous to [disapprove $\rightarrow$ _____ $\rightarrow$ chide].

Which one of the given options is appropriate to fill the blank ?

(a) grieve (b) reprise

(c) reprove (d) praise

**Sol–9: (c)**

Smile $\rightarrow$ gigle $\rightarrow$ laugh

Similarly

disapprove $\rightarrow$ **reprove** $\rightarrow$ chide

- Reprove : To criticize somebody or not approve of something that some body has done.

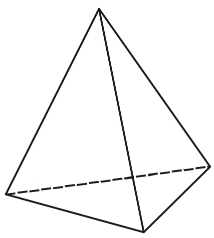
- Disapprove : To refuse approval
- Chide : criticize somebody.

**10.** Four equilateral triangle are used to form a regular closed three-dimensional object by joining along the edges. The angle between any two faces is

(a) 45° (b) 30°

(c) 60° (d) 90°

**Sol–10: (c) MTA**



In regular tetrahedron, all the faces are equilateral triangle. So, the angle (interior angle) between any two faces is 60°.

## SECTION : MECHANICAL ENGG.

**1.** Consider incompressible laminar flow over a flat plate with freestream velocity of $u_\infty$. The Nusselt number corresponding to this flow velocity is $Nu_1$. If the freestream velocity is doubled, the Nusselt number changes to $Nu_2$. Choose the correct option $Nu_2/Nu_1$.

(a) 1 (b) $\sqrt{2}$

(c) 1.26 (d) 2

**Sol–1: (b)**

$$N_{ux} = 0.332(Pr)^{1/3} \times (Re_x)^{1/2}$$

and $$Re_x = \frac{U_\infty \times H}{\nu}$$

$$Pr = \frac{\nu}{\alpha}$$

So, $$N_{ux} \propto U_\infty^{1/2}$$

$$\frac{N_{u1}}{N_{u2}} = \left(\frac{U_{\infty,1}}{2U_{\infty,1}}\right)^{1/2} \quad [\because U_{\infty 2} = 2U_{\infty 1}]$$

or $$N_{u2} = \sqrt{2} \times N_{u1}$$

**2.** A set of jobs U, V, W, X, Y, Z arrive at time t = 0 to a production line consisting of two workstations in series. Each job must be processed by both workstations in sequence (i.e. the first followed by the second). The process times (in minutes) for each job on each workstation in the production line are given below.

| Job | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|
| Workstaion 1 | 5 | 7 | 3 | 4 | 6 | 8 |
| Workstation 2 | 4 | 6 | 6 | 8 | 5 | 7 |

The sequence in which the jobs must be processed by the production line if the total make span of production is to be minimized is

(a) W-X-V-Z-Y-U

(b) W-X-Z-V-Y-U

(c) U-Y-V-Z-X-W

(d) W-U-Z-V-Y-X

**Sol–2: (b)**

By Johnson's rule of sequencing mark minimum time consuming operation for each process.

| Job | Work Stations 1 | Work Stations 2 |
|---|---|---|
| U | 5 | 4 |
| V | 7 ← | 6 |
| W | 3 | 6 |
| X | 4 | 8 |
| Y | 6 | 5 |
| Z | 8 | 7 |

According to question, work on work station (1) is followed by work on workstation (2).

Perform that job in mark station which is minimum time consumption.

It is W-X during this Z on work station-2 will be performed.

So, W – X – Z

After W-X-Z, V will be performed on workstation (1).

So, W–X–Z–V, in between Y-work on workstation will perform.

As minimum time in workstation is U, so will be performed at last.

So, W – X – Z – V – Y – U

**3.** The velocity field of a two-dimensional, incompressible flow is given by

$$V = 2\sinh x\hat{i} + v(x, y)\hat{j}$$

where $\hat{i}$ and $\hat{j}$ denote the unit vectors in x and y directions, respectively. If v(x, 0) = coshx, then v(0, –1) is

(a) 3 (b) 4

(c) 2 (d) 1

**Sol–3: (a)**

Concept:

For an incompressible flow, $\nabla \cdot \vec{V} = 0$

Given; the velocity field of a two-dimensional, incompressible flow,

$$\vec{V} = 2\sinh x\,\hat{i} + v(x,y)\hat{j} \quad \& \quad v(x,0) = \cosh x$$

Now, $\nabla \cdot \vec{V} = 0$ (for an incompressible flow)

i.e. $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$2\cosh x + \frac{\partial v}{\partial y} = 0$$

$\Rightarrow$ $\partial v = -2\cosh x \cdot \partial y$

Integrate both sides,

$$\int \partial v = -\int 2\cosh x \cdot \partial y$$

$$v = -2y\cosh x + C \quad \text{...(1)}$$

Given; v(x,0) = cosh x

$\therefore$ cosh x = –2(0) coshx + C

$\Rightarrow$ C = coshx

Now, from equation (1)

$$v(x,y) = -2y\cosh x + \cosh x = (1 - 2y)\cosh x$$

At (0, –1)

$$v(0,-1) = [1-2(-1)]\cosh(0) = 3$$

**4.** The most suitable electrode material used for joining low alloy steels using Gas Metal Arc Welding (GMAW) process is

(a) copper (b) low alloy steel

(c) cadmium (d) tungsten

**Sol–4: (b)**

For general-purpose welding of low alloy steels electrode (e.g. E7018) issued for welding of low alloy steel through GMAW.

Electrode with specific alloy compositions are selected to match base metal's composition in order to achieved desired mechanical properties in the weld.

**5.** The change in kinetic energy $\Delta E$ of an engine is 300 J, and minimum and maximum shaft speeds are $\omega_{min}$ = 220 rad/s rad/s and $\omega_{max}$ = 280 rad/s, respectively. Assume that the torque-time function is purely harmonic. To achieve a coefficient of fluctuation of 0.05, the moment of inertia (in kgm$^2$) of the flywheel to be mounted on the engine shaft is

(a) 0.071 (b) 0.096

(c) 0.113 (d) 0.053

**Sol–5: (b)**

Data given: $\Delta E$ = 300J

$\omega_{min}$ = 220 rad/sec

$\omega_{max}$ = 280 rad/sec

$C_S$ = 0.05

$C_s$ = 0.05

Total moment of inertia of system

= I

$$\omega = \frac{\omega_{max} + \omega_{min}}{2} = \frac{280 + 220}{2} = 250 \text{ rad/sec}$$

After mounting the flywheel :

$$\Delta E = (I)\omega^2 \times C_s$$

or, $300 = (I) \times (250)^2 \times 0.05$

or I = 0.096

**6.** Let f(z) be an analytic function, where z = x + iy. If the real part of f(z) is cosh x cos y, and the imaginary part of f(z) is zero for y = 0, then f(z) is

(a) coshz (b) coshz cosy

(c) coshz expz (d) coshx exp(–iy)

**Sol–6: (a)**

Concept:

The necessary & sufficient condition for a function f(z) = u(x,y) + iv(x,y) to be analytic is it should satisfy the cauchy Riemann equation.

C.R. equation:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{or,} \quad u_x = v_y$$

&, $$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \text{or,} \quad u_y = -v_x$$

Given: u = coshx cosy

Now, $u_x$ = sinhx cosy

&, $u_y$ = –coshx siny

To find f(z), put x = z & y = 0;

$$f(z) = \int (u_x - iu_y)\,dz$$

$$= \int [\sinh z \cdot \cos 0 - (-\cosh z \cdot \sin 0)]\,dz$$

$$= \int \sinh z\,dz = \cosh z + C$$

**7.** The ‘Earing’ phenomenon in metal forming is associated with

(a) rolling (b) extrusion

(c) deep drawing (d) forging

**Sol–7: (c)**

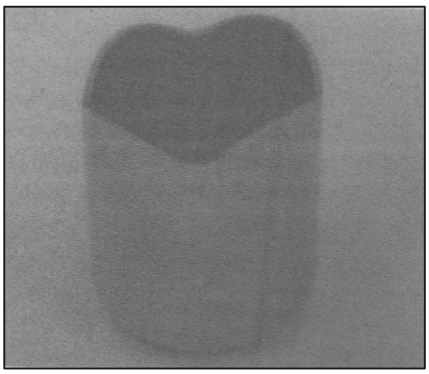
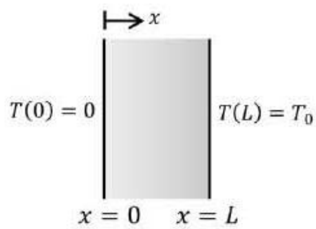


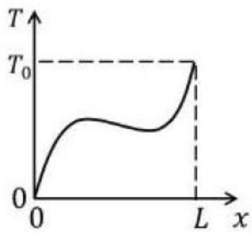
**Fig. Earning in a drawn steel cup, caused by the planar anisotropy of the sheet metal**

In deep drawing, the edges of cups may become wavy, called earing. Ears are objectionable on deep-drawn cups because they have to be trimmed off, as they serve no useful purpose, and interfere with further processing of the cup, resulting in scarp. Earing is caused by the planar anisotropy of the sheet metal, and the number of ears produced may be two, four or eight, depending on the processing history and microstructure of the material. If the sheet is stronger in the rolling direction than transverse to the rolling direction, and the strength varies uniformly with respect to orientation, then two ears will form. If the sheet has high strength at different orientations, then more ears will form.

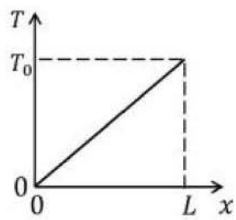
**8.** A plane, solid slab of thickness L, shown in the figure, has thermal conductivity k that varies with the spatial coordinate x as k = A + Bx, where A and B are positive constants (A > 0, B > 0). The slab walls are maintained at fixed temperature of T(x = 0) = 0 and T(x = L) = $T_0$ > 0. The slab has no internal heat sources. Considering one-dimensional heat transfer, which one of the following plots qualitatively depicts the steady-state temperature distribution within the slab ?



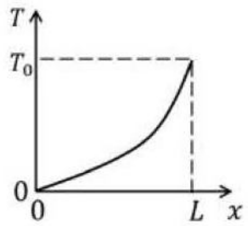

(a)



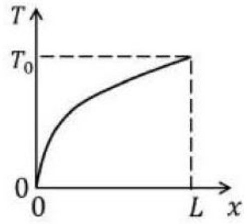

(b)



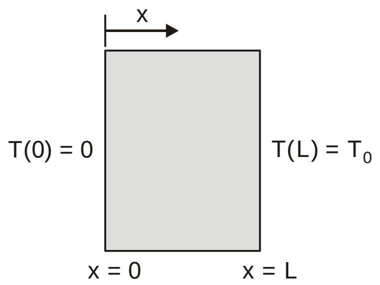
(c)




(d)




**Sol–8: (d)**




Given: $k = A + Bx$

$\dot{q}_g \to 0$

As $\dot{q} = -kA\dfrac{dT}{dx}$

or, $\dfrac{\dot{q}}{A} = C$

$= -k\left(\dfrac{dT}{dx}\right)$ for steady state

So, $-k\left(\dfrac{dT}{dx}\right) = \text{constant}$

or, $-(A+Bx)\dfrac{dT}{dx} = C$

or, $\oint_0^L dT = -\dfrac{dx \times C}{(A+Bx)}$

o $\int_0^T dT = -C\int_0^L \dfrac{dx}{A+Bx}$

So, temperature distribution will follow logarithmic curve.

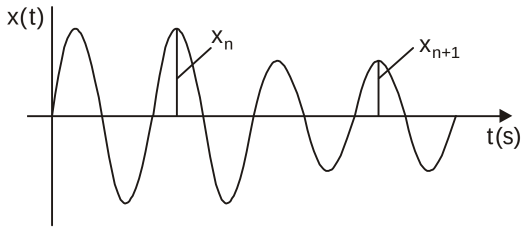
So, answer (d).

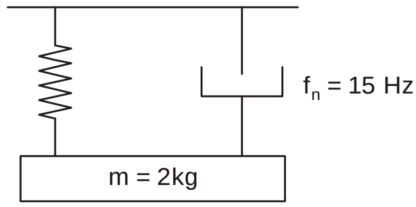
**9.** A linear spring-mass-dashpot system with a mass of 2 kg is set in motion with viscous damping. If the natural frequency is 15 Hz, and the amplitudes of two successive cycles measured are 7.75 mm and 7.20 mm, the coefficient of viscous damping (in N.s/m) is

(a) 6.11 (b) 7.51

(c) 4.41 (d) 2.52

**Sol–9: (c)**




$$x_n = 7.75 \text{ mm}$$

$$x_{n+1} = 7.20 \text{ mm}$$

$$\omega_n = \frac{2\pi}{T_n} = 2\pi(fn)$$

$$= 2\pi \times 15 = 30\pi \text{ rad/sec}$$

$$\frac{X_n}{X_{n+1}} = \frac{7.75}{7.20} = 1.07638$$

$$\delta = ln\left(\frac{x_n}{x_{n+1}}\right)$$

$$= ln(1.07638) = 0.0736$$

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

or $0.0736 = \dfrac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$

or $\varepsilon = 0.0117$

Critical damping co-efficient

$$C = \varepsilon \times 2m \times \omega_n$$

$$= 0.0117 \times 2 \times 2 \times 30\pi$$

$$= 4.41 \text{ N.s/m}$$

**10.** The grinding wheel used to provide the best surface finish is

(a) A80L5V (b) A54L5V

(c) A36L5V (d) A60L5V

**Sol–10: (a)**

**Grinding Wheel Components :**

- Abrasive Type A60-M5-V
- Abrasive size A60-M5-V
- Grade/Hardness A60-M5-V
- Structure A60-M5-V
- Bond A60-M5-V

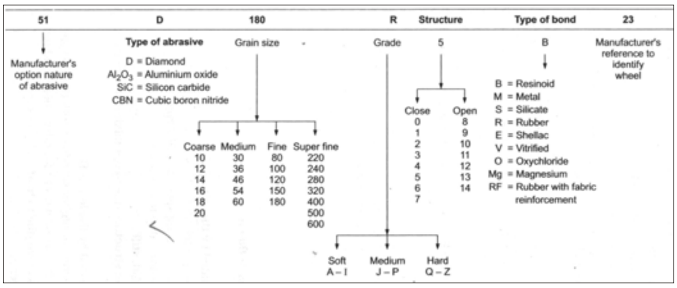



**Fig.** Specification of grinding wheel

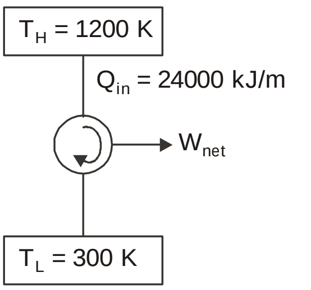
Fine and superfine grain size give best surface finish.

**11.** A furnace can supply heat steadily at 1200 K at a rate of 24000 kJ/min. The maximum amount of power (in kW) that can by using the heat supplied by the furnace in an environment at 300 K is

(a) 18000 (b) 0

(c) 300 (d) 150

**Sol–11: (c)**




$$\eta = 1 - \frac{T_L}{T_H} = \frac{W_{net}}{Q_{in}}$$

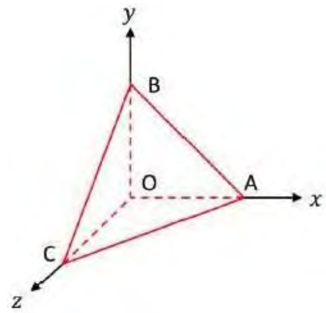
on

$$W_{net} = Q_{in}\left(1 - \frac{T_L}{T_H}\right)$$

$$= \frac{24000}{60} \times \left(1 - \frac{300}{1200}\right)$$

$$= 300 \text{ kW}$$

**12.** A rigid massless tetrahedron is placed such that vertex O is at the origin and the other three vertices A, B and C lie on the coordinate axes as shown in the figure. The body is acted on by three point loads of which one is acting at A along x-axis and another at point B along y-axis. For the body to be in equilibrium, the third point load acting at point O must be



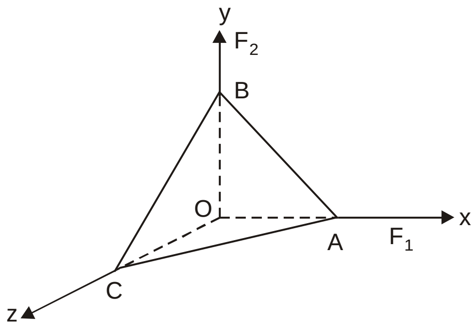

(a) along z-axis

(b) in x-y plane but not along x or y axis

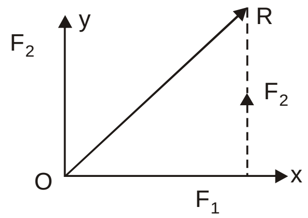
(c) in z-x plane but not along z or x axis

(d) in y-z plane but not along y or z axis

**Sol–12: (b)**




Let resultant of $F_1$ and $F_2$ is R.



For equilibrium, third force shall be in opposite direction of R in same plane (x–y plane)

So, answer is (b) in x-y plane, but not along x or y-axis.

**13.** For a ball bearing. the fatigue life in millions or revolutions is given by $L = \left(\frac{c}{P}\right)^n$, where P is the constant applied load and c is the basic dynamic load rating. Which one of the following statements is TRUE ?

(a) n = 3, assuming that the inner racing is fixed and outer racing is revolving.

(b) n = 1/3, assuming that the outer racing is fixed and the inner racing is revolving

(c) n = 3, assuming that the outer racing is fixed and inner racing is revolving

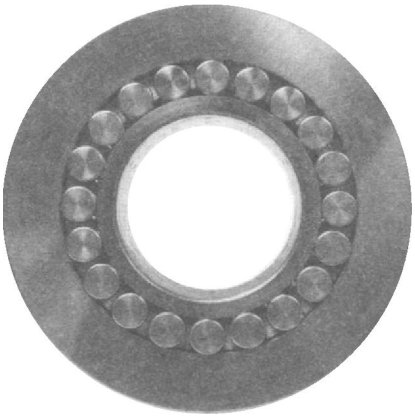
(d) n = 1/3, assuming that the inner racing is fixed and outer racing is revolving

**Sol–13: (c)**

$$L = \left(\frac{C}{P}\right)^n \times 10^6 \text{ rev.}$$

n = 3 for ball bearing

n = 10/3 for roller bearing



**14.** A queuing system has one single server workstation that admits an infinitely long queue. The rate of arrival of jobs to the queuing system follows the Poisson distribution with a mean of 5 jobs/hour. The service time of the server is exponentially distributed with a mean of 6 minutes. In steady state operation of the queuing system, the probability that the server is not busy at any point in time is

(a) 0.17 (b) 0.50

(c) 0.20 (d) 0.83

**Sol–14: (b)**

$\lambda$ = Arrival rate = 5 Jobs/hr

$\mu$ = Service rate = 6 min = 10/hr.

So, probability that the server is not busy

$$= 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{5}{10} = 1 - 0.5 = 0.50$$

**15.** Let f(.) be a twice differentiable function from $R^2 \to R$. If p, $x_0 \in R^2$ where $\|p\|$ is sufficiently small (here $\|\cdot\|$ is the Euclidean norm or distance function), then $f(x_0 + p) = f(x_0) + \nabla f(x_0)^T p + \frac{1}{2} p^T \nabla^2 f(\psi) p$ where $\psi \in R^2$ is a point on the line segment joining $x_0$ and $x_0 + p$. If $x_0$ is strict local minimum of f(x), then which one of the fallowing statements is TRUE ?

(a) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p = 0$

(b) $\nabla f(x_0)^T p > 0$ and $p^T \nabla^2 f(\psi) p = 0$

(c) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p > 0$

(d) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p < 0$

**Sol–15: (c)**

Let the function is parabolic nature,

i.e. form of : $Ax^2 + Bx + C$

Now, the conditions for strictly local minimum to exist are:

- The coefficient of $x^2$ (i.e. A) should be positive.
- First derivative of the function should be zero.
- Second derivative of the function should be positive.

Now, After the observation of all four options, above these conditions will be satisfy by option (c). Hence, option (c) will be correct.

**16.** In order to numerically solve the ordinary differential equation $\frac{dy}{dt} = -y$ for t > 0, with an initial condition y(0) = 1, the following scheme is employed

$$\frac{y_{n+1} - y_n}{\Delta t} = -\frac{1}{2}(y_{n-1} + y_n)$$

Here, $\Delta t$ is the time step and $y_n = y(n\Delta t)$ for n = 0, 1, 2, .... Thus numerical scheme will yield a solution with non-physical oscillations for $\Delta t > h$. The value of h is

(a) 1 (b) 2

(c) $\frac{3}{2}$ (d) $\frac{1}{2}$

**Sol–16: (b)**

**17.** The preparatory functions in Computer Numerical Controlled (CNC) machine programming are denoted by the alphabet

(a) M (b) O

(c) G (d) P

**Sol–17: (c)**

G-codes are also known as preparatory codes for CNC machine. The instructions provided by G-codes tell the machine tool how to move in the (X, Y, Z) Cartesian co-ordinate.

**18.** Which one of the following statements regarding a Rankine cycle is FALSE ?

(a) Superheating the steam in the boiler increases the cycle efficiency.

(b) Cycle efficiency increases as a condenser pressure decreases.

(c) Cycle efficiency increases as boiler pressure decreases.

(d) The pressure at the turbine outlet depends on the condenser temperature.

**Sol–18: (b) challenge**

Increase of efficiency :

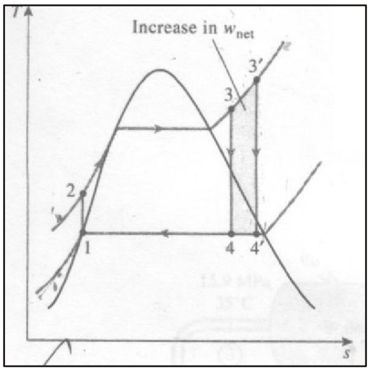



**Fig. The effect of superheating the steam to higher temperatures on the ideal Rankine cycle**

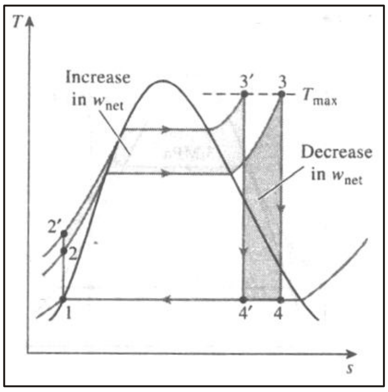



**Fig. The effect of increasing the boiler pressure on the ideal Rankine cycle.**

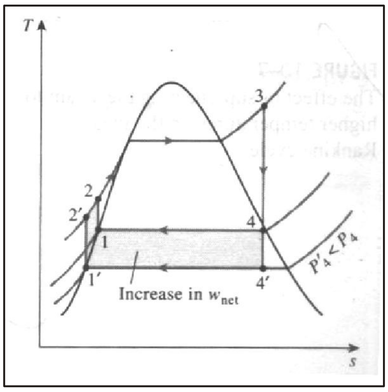



**Fig. The effect of lowering the condenser pressure on the ideal Rankine cycle.**

and pressure of condenser = f(condenser temp.)

**19.** Which one of the following failure theories is the most conservative design approach against fatigue failure ?

(a) Modified Goodman line

(b) Yield line

(c) Gerber line

(d) Soderberg line

**Sol–19: (d)**

**Gerber Line.** A parabolic curve joining $S_e$ on the ordinate to $S_{ut}$ on the abscissa is called the Gerber line.

**Soderberg Line** A straight line joining $S_e$ on the ordinate to $S_{yt}$ on the abscissas called the Soderberg line.

**Goodman Line.** A straight line joining $S_e$ on the ordinate to $S_{ut}$ on the abscissa is called the Goodman line.

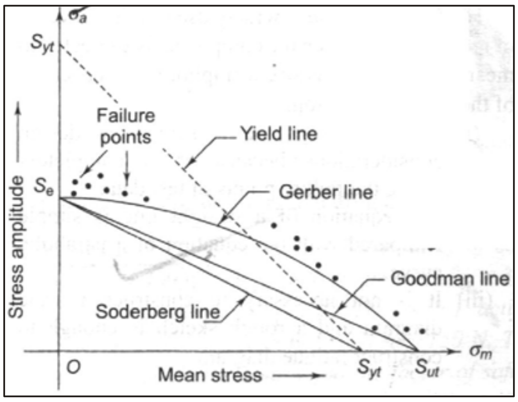



**Fig. Soderberg and Goodman line**

The Gerber parabola fits the failure points of test data in the best possible way. The goodman line fits beneath the scatter of this data. Both Gerber parabola and Goodman line intersect at ($S_e$) on the ordinate to ($S_{ut}$) on the abscissa. However, the Goodman line is more safe from design considerations because it is completely inside the Gerber parabola and inside the failure points. **The Soderberg line is a more conservative failure criterion and there is no need to consider even yielding in this case.**

**20.** Consider a hydrodynamically fully developed laminar flow through a circular pipe with the flow along the axis (i.e. z direction). In the following statements, T is the temperature of the fluid. $T_w$ is the wall temperature and $T_m$ is the bulk mean temperature of the fluid. Which one the following statements is TRUE ?

(a) Nusselt number varies linearly along the z direction for a thermally fully developed flow.

(b) For constant wall temperature of the duct, $\frac{dT_m}{dz} = \text{constant}$.

(c) For a thermal fully developed flow, $\frac{\partial T}{dz} = 0$, always.

(d) For constant wall temperature ($T_w > T_m$) of the duct, $\frac{dT_m}{dz}$ increases exponentially with distance along z direction.

**Sol–20: (d)**

In a thermally fully developed flow, the temperature profile will no longer change along length of flow, similar the velocity profile. It means there is no further growth of boundary layer. This is the case of steady state.

i.e. $$\frac{dT}{dZ} = 0$$

$N_u$ = Constant for thermally fully developed flow.

($T_w - T_m$) decays exponential

$\frac{dT_m}{d_z}$ increases exponentially

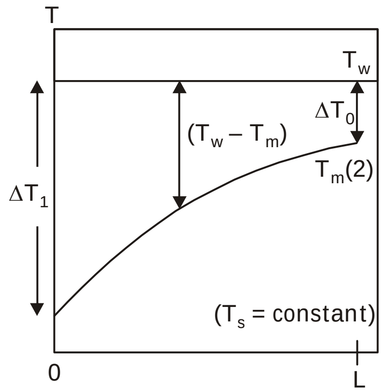
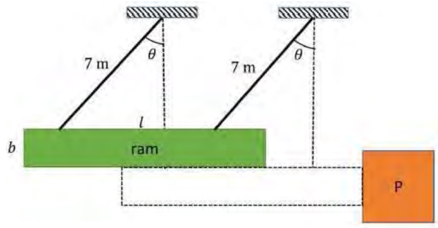



**Fig. Axial temperature variation for heat transfer in a tube for Constant surface temperature**

**21.** A ram in the form of a rectangular body of size l = 9 m and b = 2 m is suspended by two parallel ropes of lengths 7 m. Assume the center-of-mass of the body is at its geometric center and g = 9.81 m/s$^2$. For striking the object P with a horizontal velocity of 5 m/s. What is the angle $\theta$ with the vertical from which the ram should be released from rest ?



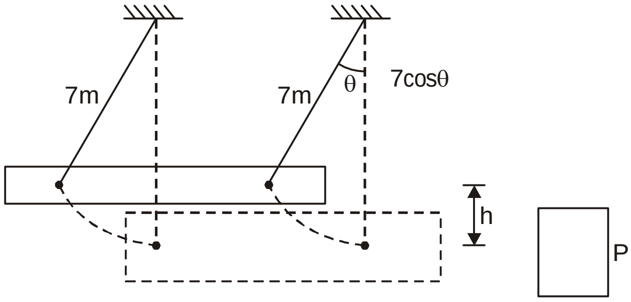

(a) 40.2°

(b) 67.1°

(c) 79.5°

(d) 35.1°

**Sol–21: (d)**




$$h = 7 - 7\cos\theta = 7(1-\cos\theta)$$

Principle of Conservation of Energy

$$\cancel{m}gh = \frac{1}{2}\cancel{m}V^2$$

$$g \times 7(1-\cos\theta) = \frac{1}{2}\times(5)^2 = 12.5$$

$$1-\cos\theta = \frac{12.5}{9.81\times 7} = 0.182$$

$$\cos\theta = 0.817$$

$$\theta = 35.1°$$

**22.** The allowance provided to a pattern for easy withdrawal from a sand mold is

(a) distortion allowance

(b) finishing allowance

(c) shake allowance

(d) shrinkage allowance

**Sol–22: (c)**

Types of allowances.

- Draft/taper allowances.
- Machining allowance/finishing allowance.
- Distortion allowances/camber allowance
- Shrinkage allowance/contraction allowances.
- Shake allowance/rapping allowance.

**Shake allowance**

Before withdrawal from the sand mould, the pattern is wrapped all around the vertical faces to enlarge the mould cavity slightly, which facilitates its removal. Since it enlarge the final casting made, it is desirable that the original pattern dimensions should be reduced to account for this increase. There is no sure way of quantifying this allowance, since it is highly dependent on the foundry personnel and practices involved.

It is negative allowance and is to be applied only to those dimensions, which are parallel to the parting plane. One way of reducing this allowance is to increase the draft, which can be removed during the subsequent matching.

**Draft/taper allowances**

Once pattern is embedded in the moulding sand now it is required to withdrawing the pattern from moulding sand, for this propose a vertical force is required to lift the pattern. For easy removal of pattern from mould cavity there should be some degree or mm taper in the pattern.

If there is no taper in the pattern then more chance to break the mould cavity wall during withdrawing the pattern. Draft allowance varies with the complexity of job.

**Machining or Finishing allowance**

In sand casting operation machining/finishing allowances is given in the pattern as on solidification every metal get shrinked up to some extend and the outer surface of object (casting) is rough, therefore, to smoothing the outer surface of casting machining allowance is must.

**Distortion allowance/Camber allowance**

On just solidification of metal casting is weak, therefore, more chances to distorted casting. Chance of distortion are basically in U, V, then long section or complicated casting. Distortion can be minimized or eliminated by providing an allowance and constructed the pattern initially distorted, i.e. opposite in outer side direction. Among of distortion allowance varies from 1.5-20 mm.

It is the tendency of all metals that they shrink after cooling except bismuth, Shrinkage is only due to inter atomic vibration which are implified by increasing in temperature. Metal shrink or contract in three ways :

- Liquid Contraction
- Solidification Contraction
- Solid Contraction

**23.** Consider the system of linear equations.

x + 2y + z = 5

2x + ay + 4z = 12

2x + 4y + 6z = b

The values of a and b such that there exists a non-trivial null space and the system admits infinite solutions are

(a) a = 8, b = 12

(b) a = 4, b = 12

(c) a = 8, b = 14

(d) a = 4, b = 14

**Sol–23: (d)**

Concept:

If $\rho(AB) = \rho(A) <$ No. of unknowns, then the system having infinite number of solutions.

Given:

$$\begin{aligned} x + 2y + z &= 5 \\ 2x + ay + 4z &= 12 \\ 2x + 4y + 6z &= b \end{aligned}$$

Augmented matrix, $\rho(AB) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 2 & a & 4 & 12 \\ 2 & 4 & 6 & b \end{array}\right]$

$R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - 2R_1$

$$\therefore \quad \rho(AB) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & a-4 & 2 & 2 \\ 0 & 0 & 4 & b-10 \end{array}\right]$$

Now, for infinite solution

$\rho(A) = \rho(AB) < 3$

i.e. $\rho(A) = \rho(AB) = 2$

Since, for $\rho(A) = 2$

a – 4 = 0

$\Rightarrow$ $\boxed{a = 4}$

&, for $\rho(AB) = 2$,

b – 10 = 4

$\Rightarrow$ $\boxed{b = 14}$

**24.** The value of the surface integral

$$\oiint_S z\,dx\,dy$$

where S is the external surface of the sphere $x^2 + y^2 + z^2 = R^2$ is

(a) 0 (b) $\pi R^3$

(c) $4\pi R^3$ (d) $\frac{4\pi}{3}R^3$

**Sol–24: (d)**

Surface Integration is defined as

$$\int_S \vec{F}\cdot\hat{n}ds = \oiint_R F_1 dydz + F_2 dxdz + F_3 dxdy$$

$$= \oiint_R zdxdy$$

This means $F_1 = 0$, $F_2 = 0$, $F_3 = z$

$\vec{F} = z\hat{K}$

$\nabla \cdot \vec{F} = 1$

Using Gauss Divergence Theorem

$$\int_S \vec{F}\cdot\hat{n}ds = \int_V \nabla\cdot\vec{F}dV = \int_V dV$$

$$= \text{Volume of surface} = \frac{4}{3}\pi R^3$$

**25.** The phase present in pearlite are

(a) martensite and ferrite

(b) ferrite and cementite

(c) austenite and ferrite

(d) cementite and austenite

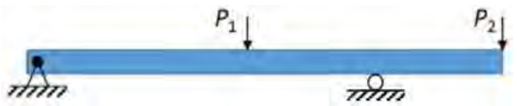
**Sol–25: (b)**

Coarse pearlite → $\alpha$ Ferrite + $Fe_3C$ → Alternating layers of a ferrite and $Fe_3C$ that are relatively thick.

Fine pearlite → $\alpha$ Ferrite + $Fe_3C$ → Alternating layers of a ferrite and $Fe_3C$ that are relatively thin.

**26.** Which of the following beam(s) is/are statically indeterminate ?

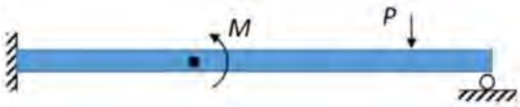
(a)




(b)


(c)


(d)




**Sol–26: (b and d)**

**27.** A liquid fills a horizontal capillary tube whose one end is dipped in a large pool of the liquid. Experiments show that the distance L travelled by the liquid meniscus inside the capillary in time t is given by

$$L = k\gamma^a R^b \mu^c \sqrt{t}$$

where $\gamma$ is the surface tension, R is the inner radius of the capillary, and $\mu$ is the dynamic viscosity of the liquid. If k is a dimensionless constant, then the exponent a is _____ (rounded off to 1 decimal place).

**Sol–27: (0.5)**

$$L = kV^a R^b \mu^c \sqrt{t}$$

Dimensions – length, L = [L]

Surface Tension, $\gamma$ = $[MT^{-2}]$

Radius, R = [L]

Dynamic viscosity, $\mu$ = $[ML^{-1}T^{-1}]$

Time, t = [T]

K is dimensionless

$$\text{So, } [L] = \left[MT^{-2}\right]^a [L]^b \left[ML^{-1}T^{-1}\right]^C \left[T^{1/2}\right]$$

$$[L] = [M]^{a+c}[L]^{b-c}[T]^{-C+\frac{1}{2}-2a}$$

$$\text{So,} \quad a + c = 0 \Rightarrow c = -a$$

$$b - c = 0$$

$$-2a - c + \frac{1}{2} = 0$$

$$\Rightarrow -2a + a + \frac{1}{2} = 0$$

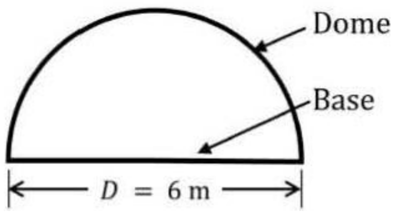
$$-a + \frac{1}{2} = 0$$

$$\Rightarrow \quad a = \frac{1}{2}$$

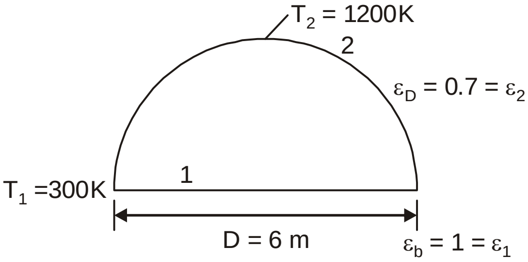
$$\Rightarrow \quad \boxed{a = 0.5}$$

**28.** Consider a hemispherical furnace of diameter D = 6m with a flat base. The dome of the furnace has an emissivity of 0.7 and the flat base is a blackbody. The base and the dome are maintained at uniform temperature of 300 K and 1200 K, respectively. Under steady state conditions, the rate of radiation heat transfer from the dome to be base is ______ kW. (rounded off to the nearest integer).

Use Stefan-Boltzmann constant = $5.67 \times 10^{-8}$ $W/(m^2K^4)$.



**Sol–28: (2726.94)**




$$F_{11} = 0$$

$$F_{12} = 1$$

From reciprocity theorem :

$$A_1F_{12} = A_2F_{21}$$

$$F_{21} = \frac{A_1F_{12}}{A_2} = \frac{\pi R^2 \times 1}{2\pi R^2} \times 1 = 0.5$$

$$Q_{net} = A_2 (F_g)_2 \times \sigma_b \times \left(T_2^4 - T_1^4\right)$$

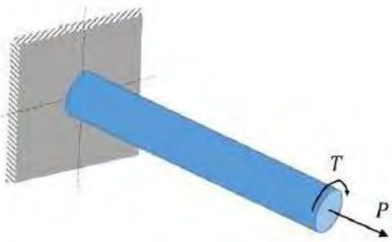
$$(Fg)_{12} = \frac{1}{\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{21}}} + \frac{1-\varepsilon}{\varepsilon_1} \times \frac{A_2}{A_1}$$

$$= \frac{1}{\frac{1-0.7}{0.7} + \frac{1}{0.5} + 0} = 0.41176$$

$$Q_{net} = 2\pi(3)^2 \times 0.41176 \times 5.67 \times 10^{-8} \times \left[1200^4 - 300^4\right]$$

$$= 2726.935 \text{ kW} \approx 2726.94 \text{ kW}$$

**29.** A solid massless cylindrical member of 50 mm diameter is rigidly attached at one end, and is subjected to an axial force P = 100 kN and a torque T = 600 N. m at the other end as shown. Assume that the axis of the cylinder is normal to the support. Considering distortion energy theory with allowable yield stress as 300 MPa, the factor of safety in the design is _____ (rounded off to 1 decimal place).




**Sol–29: (4.5295)**

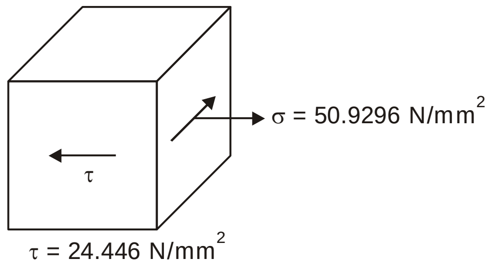
$$\text{Axial stress} = \frac{P}{A} = \frac{100 \times 10^3 \text{ N}}{\frac{\pi}{4}(50\text{mm})^2}$$

$$= 50.9296 \text{ N/mm}^2$$

$$\text{Max.shear stress} = \frac{Tr}{J}$$

$$= \frac{600\text{Nm} \times 25 \text{ mm}}{\frac{\pi (50 \text{ mm})^4}{32}} \times 10^3 \frac{\text{mm}}{\text{m}}$$

$$= 24.446 \text{ N/mm}^2$$




Principle stress are $\sigma_{max/min} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$$= \frac{50.9296}{2} \pm 56.4927$$

$$= 60.7644, -9.8348 \text{ N/mm}^2$$

As per max. distortion every theory

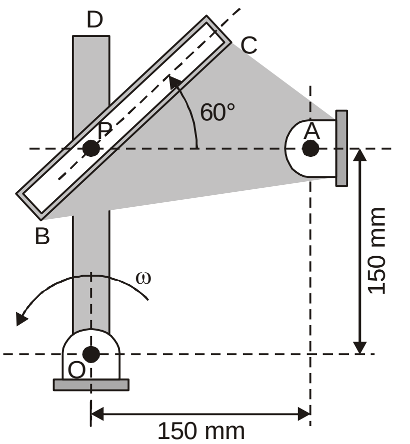
$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2\right] \le \left[\frac{f_y}{FOS}\right]^2$$

$$\frac{1}{2}\left[\begin{array}{l}(60.7644 - 9.8348)^2 + (60.7644)^2 \\ -(9.8348)^2\end{array}\right] \le \left(\frac{300}{FOS}\right)^2$$

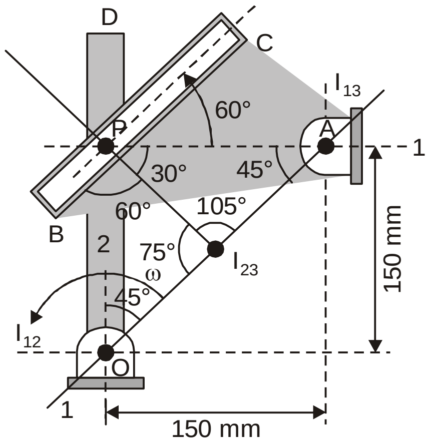
$$4386.64 \le \frac{90000}{(FOS)^2}$$

$$\Rightarrow \boxed{FOS < 4.5295}$$

**30.** At the instant when OP is vertical and AP is horizontal, the link OD is rotating counter clockwise at a constant rate $\omega = 7$ rad/s, Pin P on link OD sides in the slot BC of link ABC which is hinged at A, and cause a clockwise rotation of the link ABC. The magnitude of angular velocity of link ABC for the instant is _____ rad/s (rounded off to 2 decimal places).




**Sol–30: (12.1)**




$$V_{I23} = \omega_2 (I_{12}I_{23}) = \omega_3 (I_{13}I_{23})$$

In $\Delta I_{12} P I_{23}$

$$\frac{I_{12}I_{23}}{\sin 60} = \frac{I_{12}P}{\sin 75}$$

or $$\frac{I_{12}I_{23}}{\sin 60°} = \frac{150}{\sin 75}$$

or, $$I_{12\ 23} = 134.49 \text{ mm}$$

In $\Delta I_{13} I_{23} P$

$$\frac{I_{13}I_{23}}{\sin 30} = \frac{I_{13}P}{\sin 105}$$

$$\frac{I_{13}I_{23}}{\sin 30} = \frac{150}{\sin 105}$$

$$I_{13}I_{23} = 77.65$$

$$\omega_2 (I_{12} \times I_{23}) = \omega_3 (I_{13} \times I_{23})$$

$$7 \times 134.49 = \omega_3 \times 77.65$$

$$\omega_3 = 12.124 \approx 12.12 \text{ rad/sec}$$

**31.** A blanking operations is performed on C20 steel sheet to obtain circular disc having a diameter of 20 mm and a thickness of 2 mm. An allowance of 0.04 is provided. The punch size used for the operation is ______ mm (rounded off to 2 decimal places).

**Sol–31: (19.74)**

$$D = 20 \text{ mm}$$

$$\text{Allowance} = 0.04$$

$$\text{Thickness, } t = 2 \text{ mm}$$

For blanking operation

For C20, shear strength = 294 MPa = $\tau$

$$\text{Clearance} = 0.0032t\sqrt{\tau}$$

$$= 0.0032 \times 2 \times \sqrt{294}$$

$$= 0.1097 \text{ mm}$$

Punch Size = Dia. of blank –2×Clearance – allowance

$$\text{Punch size} = D - 2 \times C - \text{allowance}$$

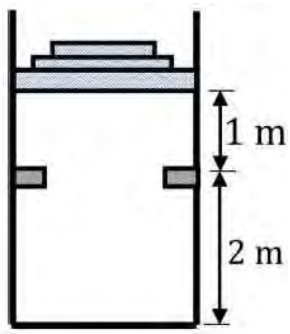
$$= 20 - 2 \times 0.1097 - 0.04$$

$$= 19.7406$$

$$= 19.74 \text{ mm}$$

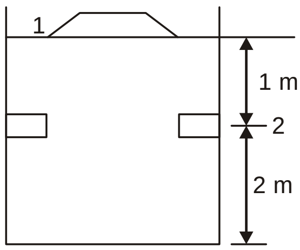
**32.** A piston-cylinder arrangement shown in the figure has a stop located 2 m above the base. The cylinder initially contains air at 140 kPa and 350°C and the piston is resting in equilibrium at a position which is 1 m above the stops. The system is now cooled to the ambient temperature of 25°C. Consider air to be an ideal gas with a value of gas constant R = 0.287 kJ(kg.K).

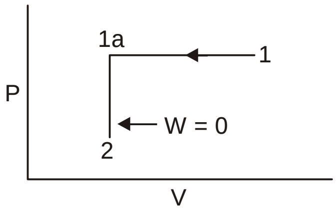
The absolute value of specific work done during the process is _____ kJ/kg (rounded off to 1 decimal place).



**Sol–32: (59.60)**

$$\text{Volume} = V_1$$
$$P_1 = 140 \text{ kPa}$$
$$T_1 = 350°C = 623 \text{ K}$$
$$T_2 = 25°C$$
$$= 273 + 25 = 298 \text{ K}$$




For constant pressure process

$$W_{1-1a} = P_a V_a - P_1 V_1$$

$P_1 = P_a$ for constant pressure process.

So, $$W_{1-1a} = mR(T_1 - T_a)$$

and $$\frac{T_a}{T_1} = \frac{V_a}{V_1}$$

or $$\frac{T_a}{T_1} = \frac{2}{3}$$

[as the stopper, length is 2 m area is constant]

an $$\frac{T_a}{623} = \frac{2}{3}$$

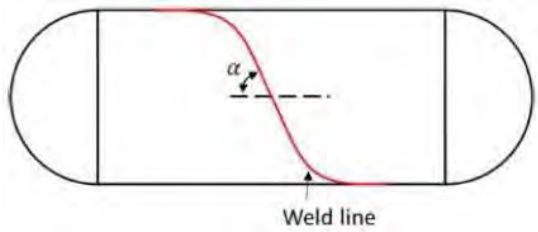
$$T_a = 415.33 \text{ K}$$

So, $$W_{1-1a} = 1 \times 0.287(623 - 415.33)$$
$$= 59.60 \text{ kJ/kg}$$

For $l_{a-2}$, $W \to 0$

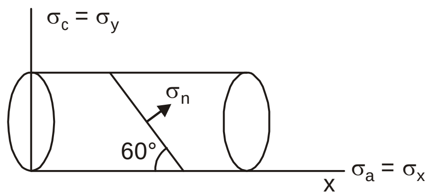
Total work done = 59.60 kJ/kg

**33.** The figure shows a thin cylinder pressure vessel constructed by welding plates together along a line that makes an angle $\alpha = 60°$ with the horizontal. The closed vessel has a wall thickness of 10 mm and diameter of 2 m. When subjected to an internal pressure of 200 kPa, the magnitude of the normal stress acting on the weld is ______ MPa (rounded off to 1 decimal place).




**Sol–33: (12.5)**

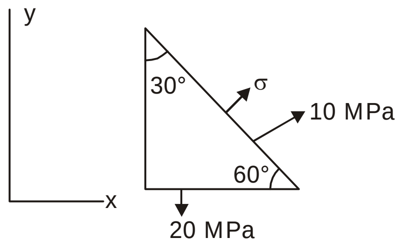
$$P = 200 \text{ kPa} = 0.2 \text{ MPa}$$
$$d = 2 \text{ m}$$
$$t = 10 \text{ mm}$$




$$\sigma_a = \sigma_l = \frac{Pd}{4t}$$

$$= \frac{0.2 \times 2 \times 10^3}{4 \times 10} = 10 \text{ MPa}$$

$$\Rightarrow \sigma_C = \sigma_y = -2\sigma_l = 2 \times \sigma_x = 2 \times 10 = 20 \text{ MPa}$$

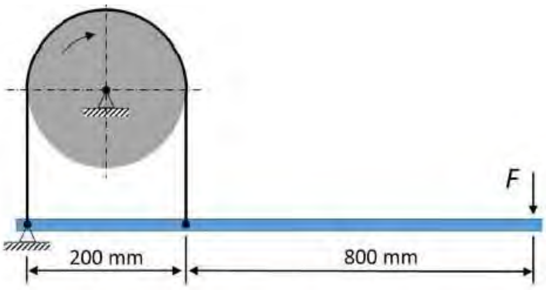



$$\sigma_0 = \sigma_n$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \frac{(\sigma_x - \sigma_y)}{2}\cos 2 \times 30° + \tau_{xy} \times \sin 2 \times 30°$$

$$\sigma_n = \frac{10 + 20}{2} + \frac{(10 - 20)}{2}\cos 60 + 0 = 12.5 \text{ MPa}$$

**34.** A band brake shown in the figure has a coefficient of friction of 0.3. The band can take a maximum force of 1.5 kN. The maximum braking force (F) that can be safely applied is _____ N. (rounded off the nearest integer).



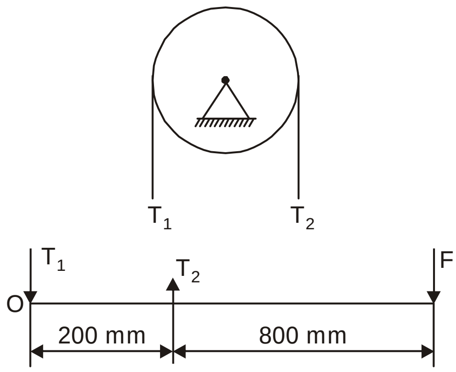

**Sol–34: (117)**

Data given, $\mu = 0.3$

$F = 1.5kN = 1500\text{ N}$

$\theta = 180° = \pi$

F.B.D. of system is




$$\sum M_0 = 0$$

$$F \times 1000 - T_2 \times 200 = 0$$

$$F = \frac{T_2}{5}$$

and $$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1500}{T_2} = e^{0.3\times\pi}$$

$$T_2 = 584.492\text{ N}$$

So, $$F = \frac{T_2}{5} = \frac{584.432}{5}$$

$$= 116.9\text{ N} \approx 117\text{ N}$$

**35.** Aluminium is casted in a cube-shaped mold having dimensions as 20 mm × 20 mm × 20 mm. Another mold of the same mold material is used to cast a sphere of aluminium having a diameter of 20 mm. The pouring temperature for both cases is the same. The ratio of the solidification times of the cube-shaped mold to the spherical mold is _____ (answer in integer).

**Sol–35: (1)**

Given, D = 20 mm for sphere

Volume of cube = 20 × 20 × 20 mm$^2$

According chvorinov's rule

$$t \propto \left(\frac{V}{A}\right)^2$$

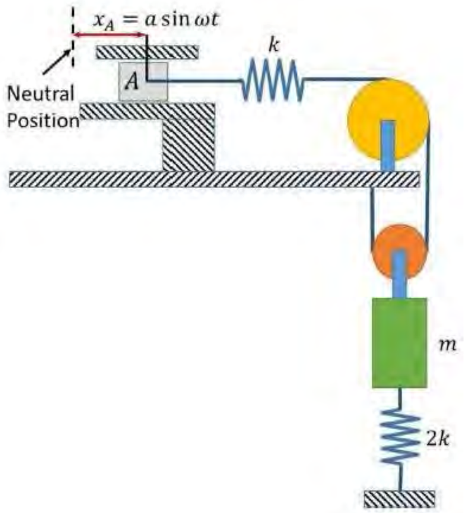
$$\frac{t_{cube}}{t_{sphere}} = \frac{\left(\frac{V_{cube}}{A_{cube}}\right)^2}{\left(\frac{V_{sphere}}{A_{sphere}}\right)^2}$$

$$= \frac{\left(\frac{20\times20\times20}{6\times20^2}\right)^2}{\left(\frac{\frac{\pi D^3}{6}}{\pi D^2}\right)^2}$$

$$= \left(\frac{20\times6}{6\times D}\right)^2$$

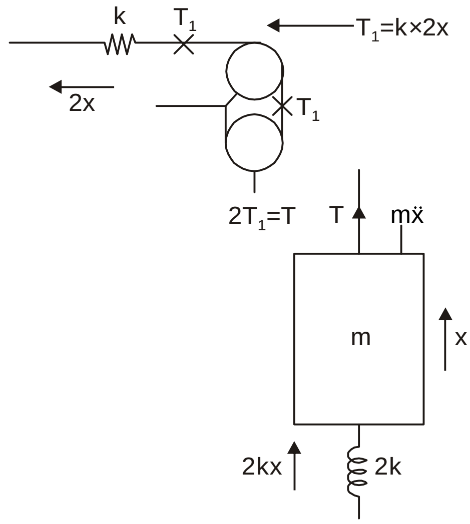
$$= \left(\frac{20}{D}\right)^2 = \left(\frac{20}{20}\right)^2 = 1$$

**36.** A vibratory system consists of mass m, a vertical spring of stiffness 2k and a horizontal spring of stiffness k. The end A of the horizontal spring is given a horizontal motion $x_A = a\sin\omega t$. The other end of the spring is connected to an inextensible rope that passes over two massless pulleys as shown. Assume m = 10 kg, k = 1.5 kN/m, and neglect friction. The magnitude of critical driving frequency for which the oscillations of mass m tend to become excessively large is _____ rad/s (answer in integer).




**Sol–36: (30)**

FBD is as follow :




$$T_1 = 2kx$$

$$T = 2T_1$$

Note : If mass 'm' move by x, then block A by 2x.

Data given :

$$m = 10 \text{ Kg}$$

$$k = 1.5 \text{ kN/m} = 1500 \text{ N/m}$$

$$\omega_n = ?$$

By D'Alembert's principle to mass m

$$m\ddot{x} + 4kx + 2kx = 0$$

$$\ddot{x} + \left(\frac{6k}{m}\right)x = 0$$

$$\omega_n = \sqrt{\frac{6k}{m}} = \sqrt{\frac{6 \times 1500}{10}}$$

$$= 30 \text{ rad/sec.}$$

**37.** In a supplier-retailer supply chain, the demand of each retailer, the capacity of each supplier, and the unit cost is rupees of material supply from each supplier to each retailer are tabulated below. The supply chain manager wishes to minimize the total cost of transportation across the supply chain.

| | Reailer I | Retailer II | Retailer III | Retailer IV | Capacity |
|---|---|---|---|---|---|
| Supplier A | 11 | 16 | 19 | 13 | 300 |
| Supplier B | 5 | 10 | 7 | 8 | 300 |
| Supplier C | 12 | 14 | 17 | 11 | 300 |
| Supplier D | 8 | 15 | 11 | 9 | 300 |
| Demand | 300 | 300 | 300 | 300 | |

The optimal cost of satisfying the total demand from all retailer is _____ rupees (answer in integer).

**Sol–37: (12300)**

By Hungarian method

| | | | | | Capacity |
|---|---|---|---|---|---|
| | 11 | 16 | 19 | 13 | 300 |
| | 5 | 10 | 7 | 8 | 300 |
| | 12 | 14 | 17 | 11 | 300 |
| | 8 | 15 | 11 | 9 | 300 |
| Demand | 300 | 300 | 300 | 300 | |

For row :

| | | | |
|---|---|---|---|
| 6 | 6 | 12 | 5 |
| 0 | 0 | 0 | 0 |
| 7 | 4 | 10 | 3 |
| 3 | 5 | 4 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 7 | 0 |
| 0 | 0 | 0 | 0 |
| 4 | 1 | 7 | 0 |
| 2 | 4 | 3 | 0 |

⇕

| | | | |
|---|---|---|---|
| 0 | 0 | 6 | 0 |
| 0 | 0 | 0 | 0 |
| 3 | 0 | 6 | 0 |
| 1 | 3 | 2 | 0 |

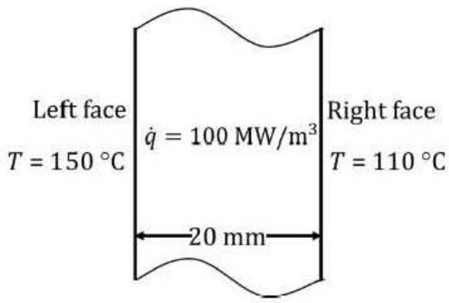
As minimum no. of lines is equal to order of matrix and this is optimal because each row and column precisely are encircled zero.

| | | | |
|---|---|---|---|
| $\boxed{0}$ | 0 | 6 | 0 |
| 0 | 0 | $\boxed{0}$ | 0 |
| 3 | $\boxed{0}$ | 6 | 0 |
| 1 | 3 | 2 | $\boxed{0}$ |

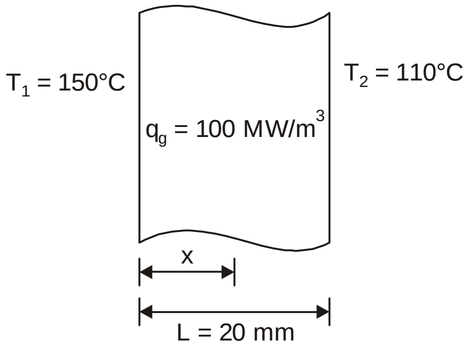
Optimal solution = (11 + 7 + 14 + 9) × 300

= 12300

**38.** Consider a slab of 20 mm thickness. There is a uniform heat generation of $\dot{q}$ = 100 MW/m$^3$ inside the slab. The left and right faces of the slab are maintained at 150°C and 110°C, respectively. The plate has a constant thermal conductivity of 200W/(m.K). Considering a 1-D steady state heat conduction, the location of the maximum temperature from the left face will be at _____ mm (answer in integer).




**Sol–38: (6)**




$$k = 200 \text{ W/mK}$$

When temperature of both wall is different in that case

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_g}{k} = 0$$

For maximum temperature

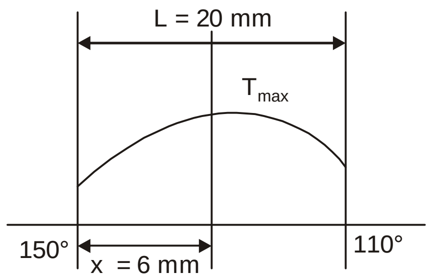
$$\frac{x}{L} = \frac{M-1}{2M}$$

where, $$M = \frac{\dot{q}_g L^2}{2k(T_1 - T_2)}$$

$$= \frac{100 \times 20^2 \times 10^{-6} \times 10^6}{2 \times 200(150-110)} = 2.5$$

So, $$x = \frac{2.5-1}{2.5 \times 2} \times 20$$

$$= 6 \text{ mm}$$



**39.** At the current basic feasible solution (bfs) $v_0\left(v_0 \in R^5\right)$, the simplex method yields the following form of a linear programming problem in standard form.

$$\begin{array}{ll} \text{minimize} & z = -x_1 - 2x_2 \\ \text{s.t.} & x_3 = 2 + 2x_1 - x_2 \\ & x_4 = 7 + x_1 - 2x_2 \\ & x_5 = 3 - x_1 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

Here the objective function is written as a function of the non-basic variables. If the simplex method moves to the adjacent bfs $v_1\left(v_1 \in R^5\right)$ that best improves the objective function, which of the following represents the objective function at $v_1$, assuming that the objective function is written in the same matter as above ?

(a) $z = -6 - 5x_1 + 2x_3$

(b) $z = -3 + x_5 - 2x_2$

(c) $z = -4 - 5x_1 - 2x_4$

(d) $z = -4 - 5x_1 + 2x_3$

**Sol–39: (d)**

$z = -x_1 - 2x_2$ ....(A)

$x_3 = 2 + 2x_1 - x_2$ ....(i)

$x_4 = 7 + x_1 - 2x_2$ ....(ii)

$x_5 = 3 - x$ ....(iii)

and $x_1, x_2, x_3, x_4, x_5 \geq 0$

From eq. (i)

$x_2 = 2 + 2x_1 - x_3$

Putting this value in the objective function eq. (A)

$z = -x_1 - 2\left(2 + 2x_1 - x_3\right)$

$= -4 - 5x_1 + 2x_3$

From eq. (ii)

$x_1 = x_4 + 2x_2 - 7$

Put these value in eq. (A)

$z = -\left(x_4 + 2x_2 - 7\right) - 2x_2$

$= 7 - 4x_2 - x_4$

From eq. (iii)

$x_1 = 3 - x_5$

Put these value in eq. (A)

$z = -\left(3 - x_5\right) - 2x_2$

$= x_5 - 3 - 2x_2$

(b) and (d) both are correct but question type is mcq so go for option (d).

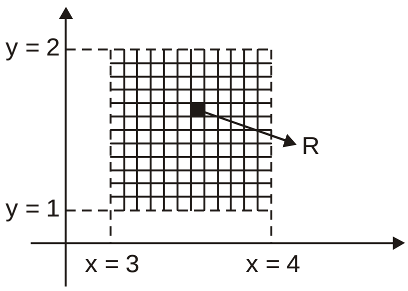
**40.** If the value of the double integral

$$\int_{x=3}^{4} \int_{y=1}^{2} \frac{dydx}{(x+y)^2}$$

is $\log_e(a/24)$, then a is _____ (answer in integer).

**Sol–40: (25)**

$$I = \int_{x=3}^{4} \int_{y=1}^{2} \frac{dydx}{(x+y)^2}$$




$$= \int_{x=3}^{4} \left( \int_{y=3}^{2} \frac{dy}{(x+y)^2} \right) dx$$

$$= \int_{x=3}^{4} \left( \int_{x+1}^{x+2} \frac{dt}{t^2} \right) dx$$

(assume x + y =t , dy = dt)

$$= \int_{x=3}^{4} \left( \frac{t^{-2+1}}{-2+1} \right)_{x+1}^{x+2} dx$$

$$= \int_{3}^{4} \left( \frac{1}{x+1} - \frac{1}{x+2} \right) dx$$

$$= \left( \ln(x+1) - \ln(x+2) \right)_3^4$$

$$= (\ln 5 - \ln 6) - (\ln 4 - \ln 5)$$

$$= \ln\left( \frac{25}{24} \right)$$

$$\log_e\left(\frac{a}{24}\right) = \log_e\left(\frac{25}{24}\right)$$

$$a = 25$$

**41.** In an arc welding process, the voltage and current are 30 V and 200A, respectively. The cross-sectional area of the joint is 20 $mm^2$ and the welding speed is 5 mm/s. The heat required to melt the material is 20 J/s. The percentage of heat lost to the surrounding during the welding process is _____ (rounded off to 2 decimal places).

**Sol–41: (66.6)**

Data given, $I = 200A$

$V = 30,\ u = 5$ mm/sec

$A = 20\ mm^2$

Heat required to melt = 20 J/sec = heat rate

Heat required for melting

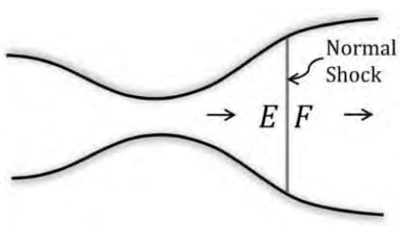
= A × u × Heat rate

= 20 × 5 × 20 = 2000W

Power, VI = 30 × 200 = 6000W

$$\%\text{ of heat generation} = \frac{6000-2000}{6000}\times 100 = 66.6\%$$

**42.** Steady, compressible flow of air takes place through an adiabatic converging diverging nozzle, as shown in the figure. For a particular value of pressure difference across the nozzle, a stationary normal shock wave forms in the diverging section of the nozzle. If E and F denote the flow conditions just upstream and downstream of the normal shock, respectively, which of the following statement(s) is/are TRUE ?




(a) Mach number of E is lower than the Mach number at F.

(b) Static pressure at E is lower than the static pressure at F.

(c) Density at E is lower than the density at F.

(d) Specific entropy at E is lower than the specific entropy at F.

**Sol–42: (b,c,d)**

For shock, flow must change from supersonic to subsonic

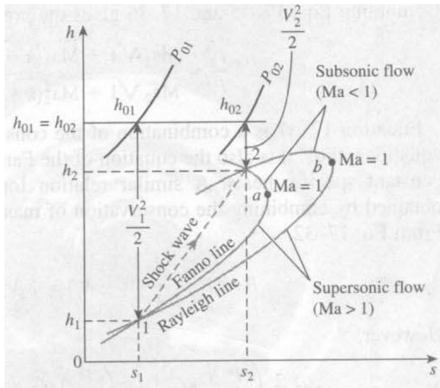



**Fig.** The h-s diagram for flow across a normal shock

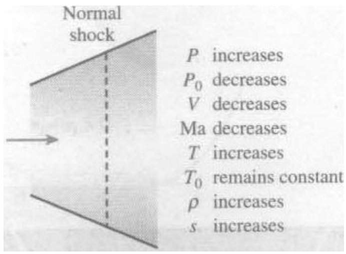



**Fig.** Variation of flow properties across a normal shock

**43.** Let X be a continuous random variable defined on [0, 1] such that its probability density function f(x) = 1 for $0 \le x \le 1$ and 0 otherwise. Let Y = $\log_e(X + 1)$. Then the expected value of Y is _____. (rounded off to 2 decimal places).

**Sol–43: (0.39)**

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$y = \ln(x + 1)$$

$$E(x) = \int x\, f(x)\, dx$$

$$E(g(x)) = \int g(x)\, f(x)\, dx$$

$$E(y) = \int \ln(x+1)\, f(x)\, dx$$

$$E(y) = \int_0^1 \ln(x+1)\, dx \quad \{f(x) = 1, 0 < x < 1\}$$

$$E(y) = \left(x \ln(x+1) - \int_0^1 \frac{x}{x+1}\,dx\right)_0^1$$

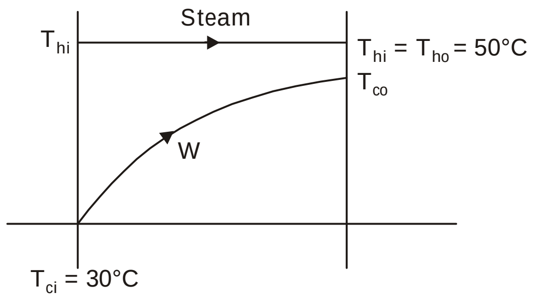
$$= \left(x \ln(x+1) - [x - \ln(x+1)]\right)_0^1$$

$$= \left(\ln(x+1)(x+1) - x\right)_0^1$$

$$= 2 \ln 2 - 1 = 0.38629 = 0.3863 \simeq 0.39$$

**44.** A condenser is used as a heat exchanger in a large steam power plant in which steam is condensed to liquid water. The condenser is a shell and tube heat exchanger which consists of 1 shell and 20,000 tubes. Water flows through each of the tubes at a rate of 1 kg/s with an inlet temperature of 30°C. The steam in the condenser shell condenses at the rate of 430 kg/s at a temperature of 50°C. If the heat of vaporization is 2.326 MJ/kg and specific heat of water is 4 kJ/(kg.K), the effectiveness of the heat exchanger is _______ (rounded off to 3 decimal places).

**Sol–44: (0.625)**




$$\text{no. of tube, } n = 20{,}000$$

$$\dot{m}_w = 1 \text{ kg/sec/per tube}$$

$$\dot{m}_s = 430 \text{ kg/sec}$$

So,

$$\dot{M}_w = n \times \dot{m}_w$$

$$= 20000 \times 1 = 20000 \text{ kg/sec}$$

$$LHV = 2.236 \text{ MJ/kg}$$

$$C = 4 \text{ kJ/kgK}$$

$$C_{min} = \dot{M}_w \times C = 20000 \times 4$$

$$\varepsilon = \frac{\dot{m}_s \times LHV}{C_{min}(T_{h_i} - T_{c_i})}$$

$$= \frac{430 \times 2.326 \times 10^6}{20000 \times 4(50-30)}$$

$$\varepsilon = 0.625$$

**45.** The matrix $\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}$ (where a > 0) has a negative eigenvalue if a is grater than

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{1}{5}$

**Sol–45: (c)**

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & a \\ 8 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(3-\lambda) - 8 = 0$$

$$\lambda^2 - 4\lambda + 3 - 8a = 0$$

$$\lambda_1 \times \lambda_2 = \frac{3-8a}{1} \text{ (product of roots)}$$

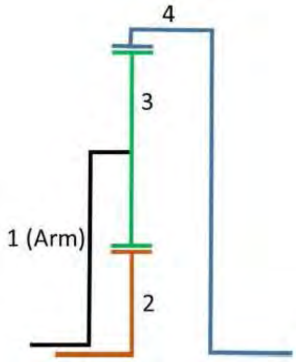
$$= 3 - 8a$$

If one eigen value is negative then product of eigen values must be negative.

$$3 - 8a < 0$$

$$a > 3/8$$

**46.** The Levai type-A train illustrated in the figure has gears with module m = 8 mm/tooth. Gears 2 and 3 have 19 and 24 teeth respectively. Gear 2 is fixed and internal gear 4 rotates at 20 rev/min counter-clockwise. The magnitude of angular velocity of the arm is _____ rev/min. (rounded off to 2 decimal places).



**Sol–46: (15.58)**

Data given $m = 8$ mm

$T_2 = 19$

$T_3 = 24$

$N_2 = 0$

$N_4 = 20$ rpm

| Conditions of motion | Arm | Gear 2 | Gear 3 | Gear 4 |
|---|---|---|---|---|
| Arm fixed-gear C rotates through + 1 revolution (i.e., 1 rev. anticlockwise) | 0 | +1 | $-\frac{T2}{T_3}$ | $-\frac{T_2}{T_3}\times\frac{T_3}{T_4}=-\frac{T_2}{T_4}$ |
| Arm fixed-gear C rotates through + x revolutions | 0 | +x | $-x\times\frac{T2}{T_3}$ | $-x\times\frac{T_2}{T_4}$ |
| Add + y revolutions to all elements | +y | +y | +y | +y |
| Total motion | +y | x + y | $y-x\times\frac{T2}{T_3}$ | $y-x\times\frac{T_2}{T_4}$ |

$$r_4 = r_2 + 2r_3$$

$$\frac{mT_4}{2} = \frac{mT_2}{2}+\frac{2m\times T_3}{2}$$

or, $T_4 = T_2 \times 2T_3$

i.e., $T_4 = 19 + 2 \times 24 = 67$

As $N_2 = 0$

So, $x + y = 0$ ...(i)

$$N_4 = 20 = y-x\times\frac{19}{67}$$

or, $y-x\times\frac{19}{67} = 20$ ...(ii)

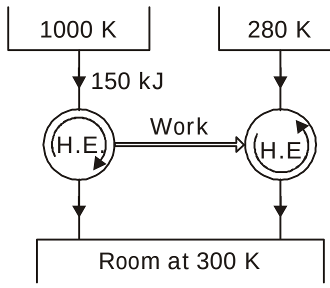
By (i) and (ii)

$k = –15.58$ rpm

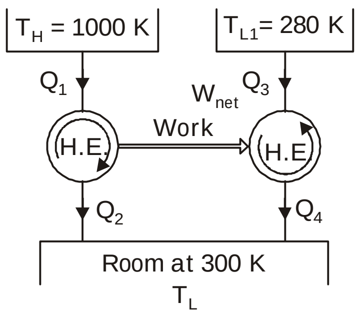
$= 15.58$ clockwise

$y = 15.58$ counterclockwise

**47.** A heat pump (H.P.) is driven by the work output a heat engine (H.E.) as shown in the figure. The heat engine extracts 150 kJ of heat from the source at 1000 K. The heat pump absorbs heat from the ambient at 280 K and delivers heat to the room which is maintained at 300 K. Considering the combined system to be ideal, the total amount of heat delivered to the room together by the heat engine and heat pump is ____ kJ (answer in integer).




**Sol–47: (1620)**




$$\eta_{HE} = 1-\frac{T_L}{T_H}=1-\frac{300}{1000}=1-\frac{Q_2}{Q_1}$$

or, $Q_2 = 0.3Q_1 = 0.3 \times 150 = 45$ kJ

$W_{net} = Q_1 – Q_2 = 150 – 45 = 105$ kJ

$$(COP)_{HP} = \frac{Q_4}{W_{net}}=\frac{T_L}{T_L-T_{L1}}$$

or, $$\frac{Q_4}{105} = \frac{300}{300-280}$$

or, $Q_4 = 1575$ kJ

So, net heat delivered to room

$Q_4 + Q_2 = 1575 + 45 = 1620$ kJ

**48.** If x(t) satisfies the differential equation.

$$t\frac{dx}{dt}+(t-x)=0$$

subject to the condition x(1) = 0, then the value of x(2) is ____. (rounded off to 2 decimal places).

**Sol–48: (–1.3863)**

$$t\frac{dx}{dt}+t-x = 0$$

$$\frac{dx}{dt}+1-\frac{x}{t} = 0$$

$$\frac{dx}{dt}-\frac{x}{t} = -1 \quad ...(i)$$

Comparing given O.D.E with

$$\frac{dx}{dt}+px = Q$$

$$P = -\frac{1}{t},\ Q = -1$$

$$\text{I.F.} = e^{\int -\frac{1}{t}dt}$$

$$= e^{-ln\,t}$$

$$= e^{ln\,1/t}$$

$$= \frac{1}{t}$$

Solution of (i) is given by

$$x \times \text{I.F.} = \int \text{I.F.}\,Q dt + C$$

$$x \times \frac{1}{t} = \int -\frac{1}{t}dt + C$$

$$\frac{x}{t} = -ln t + C$$

$$\frac{0}{1} = -ln 1 + C \text{ (using initial condition)}$$

$$C = 0$$

$$\frac{x}{t} = -ln t \Rightarrow x - t\ ln t$$

$$x(2) = -2ln 2 = -1.38629 = -1.3863$$

$$= -1.3863$$

**49.** A cutting tool provides a tool life of 60 minutes while machining with the cutting speed of 60 m/min. When the same tool is used for machining the same material, it provides a tool life of 10 minutes for a cutting speed of 100 m/min. If the cutting speed is changed to 80 m/min for the same tool and work material combination, the tool life computed using Taylor's tool life model is _____ minutes (rounded off to 2 decimal places).

**Sol–49: (21.87)**

Taylor Tool Life Equation

$$VT^n = \text{Constant}$$

$$V_1T_1^n = V_2T_2^n = V_3T_3^n$$

$$V_1T_1^n = V_2T_2^n$$

Given : $V_1 = 60\frac{m}{min},\ T_1 = 60$ min

$V_2 = 100\frac{m}{min},\ T_2 = 10$ min

So, to find out n, $V_1T_1^n = V_2T_2^n$

$$lnV_1 + n\,lnT_1 = lnV_2 + n\,lnT_2$$

$$lnV_1 - lnV_2 = n[lnT_2 - lnT_1]$$

$$n = \frac{lnV_1 - lnV_2}{lnT_2 - lnT_1}$$

$$= \frac{ln(60) - ln(100)}{ln(10) - ln(60)} = 0.285$$

Now, $V_3 = 80$ m/min, $T_3 = ?$

$$V_3T_3^n = V_2T_2^n$$

So, $80 \times T_3^{0.285} = 100 \times 10^{0.285}$

$\Rightarrow$ $T_3 = 21.87$ min

**50.** A flat surface of a C60 steel having dimensions of 100 mm (length) × 200 mm (width) is produced by a HSS slab mill cutter. The 8-toothed cutter has 100 mm diameter and 200 mm width. The feed per tooth is 0.1 mm, cutting velocity is 20 m/min and depth of cut is 2 mm. The machining time required to remove the entire stock is _____ minutes (rounded off to 2 decimal places).

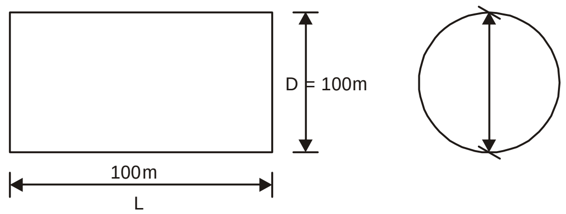
**Sol–50: (2.24)**

Data given:

$Z = 8,\ D = 100$mm, $w = 200$mm

$V = 20$m/min, $d = 2$mm, $f_t = 0.1$mm/tooth

$N = ?$



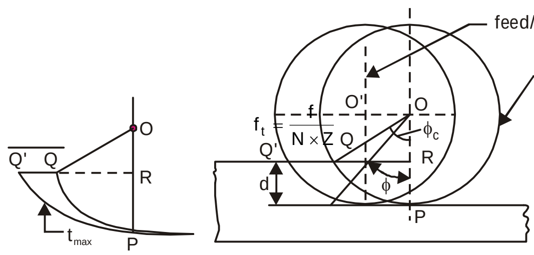




Fig. Cross-section of chip generate by milling

$QR = \sqrt{d(D-d)} = \sqrt{2\times(100-2)} = 14mm$

Le = effective length of approach = QR + L

= 14 + 100 = 114 mm

= For removal of entire length

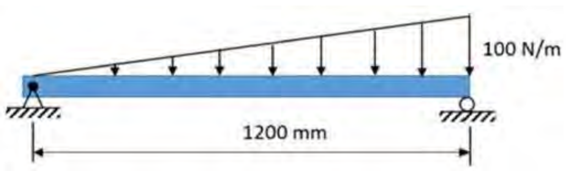
$$N = \frac{V\times1000}{\pi D} = \frac{20\times1000}{\pi\times100} = 63.662 rpm$$

So, feed = $f_t$ × NZ = 0.1 × 63.662 × 8

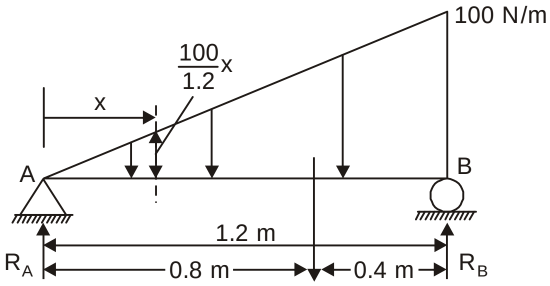
= 50.93 mm/min

$$t_{m/c} = \frac{Le}{F} = \frac{114}{50.93} = 2.24 \text{ min}$$

**51.** A horizontal beam of length 1200 mm is pinned at the left end and is resting on a roller at the other end as shown in the figure. A linearly varying distributed load is applied on the beam. The magnitude of maximum bending moment acting on the beam is ____ N.m. (round off to 1 decimal place).




**Sol–51: (9.2376)**




Net load, W = 1/2 × 100 × 1.2 = 60 kN

$$\sum M_A = 0$$

$$R_B \times 1.2 - 60\times0.8 = 0$$

$$\Rightarrow R_B = \frac{60\times0.8}{1.2} = \frac{60\times2}{3} = 40 \text{ kN}$$

$$\sum F_V = 0$$

$$R_A + R_B = 60$$
$$R_A = 20 \text{ kN}$$

BM at a distance 'x' from end (A) = $M_x$

$$M_x = \left[R_A\times x - \frac{1}{2}\times\left(\frac{100}{1.2}x\right)\times x\right]\times\frac{x}{3}$$

$$M_x = 20x - \frac{100}{7.2}x^3$$

For $M_x$ to be max.,

$$\frac{dM_x}{dx} = 0$$

$$\Rightarrow 20 - \frac{3\times100x^2}{7.2} = 0$$

$$\Rightarrow x = \sqrt{\frac{20\times7.2}{300}} = 0.6928 \text{ m}$$

$$M_{max} = 20(0.6928) - \frac{100}{7.2}(0.6928)^3 = 9.2376 \text{ Nm}$$

**52.** A company orders gears in conditions identical to those considered in the economic order quantity (EOQ) model in inventory control. The annual demand is 8000 gears, the cost per order is 300 rupees, and the holding cost is 12 rupees per month per gear. The company uses an order size that is 25% more than the optimal order quantity determined by the EOQ model. The percentage change in the total cost of ordering and holding inventory from that associated with the optimal order quantity is

(a) 12.5 (b) 2.5

(c) 0 (d) 5

**Sol–52: (b)**

Data given:

D = 8000 gear/year

$C_0$ = 300

$C_h$ = 12/month = 144/year

$$Q_{actual} = \left(1+\frac{25}{100}\right)EOQ = 1.25EOQ$$

$$EOQ = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times8000\times300}{144}} = 182.5$$

$Q_{actual}$ = 1.25 × EOQ = 1.25 × 182.5

= 1.25 × 182.5 = 228.125

Total cost at EOQ

$$TIC_1 = \sqrt{2\times C_0\times C_h\times D}$$

$$= \sqrt{2\times 300\times 144\times 8000} = 26920.4$$

Total cost at actual order ($Q_{actual}$)

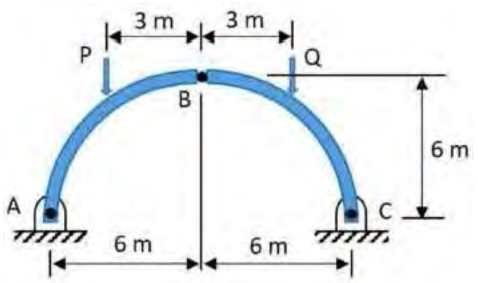
$$TIC_2 = \frac{Q_{actual}}{2}\times C_h + \frac{D}{Q_{actual}}\times C_0$$

$$= \frac{228.125}{2}\times 144 + \frac{8000}{228.125}\times 300$$

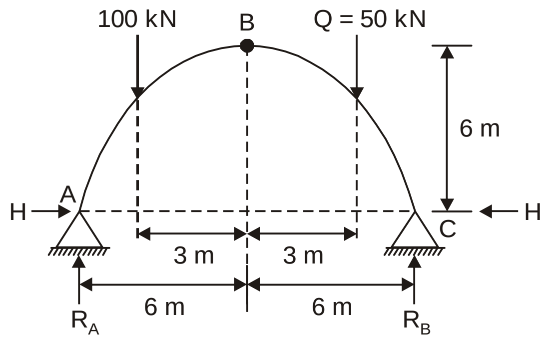
$$= 26945.54$$

$$\% \uparrow = \frac{26945.54 - 26290.68}{26290.68}\times 100 = 2.5\%$$

**53.** A three-hinge arch ABC in the form of a semi-circle is shown in the figure. The arch is in static equilibrium under vertical loads of P = 100 kN and Q = 50 kN. Neglect friction at all the hinges. The magnitude of the horizontal reaction at B is _____ kN (rounded off to 1 decimal place).




**Sol–53: (37.5)**




$$\sum F_V = 0 \Rightarrow R_A + R_C = 150 \text{ kN}$$

$$\sum M_A = 0$$

$$\Rightarrow -R_C\times 12 + 50\times 9 + 100\times 3 = 0$$

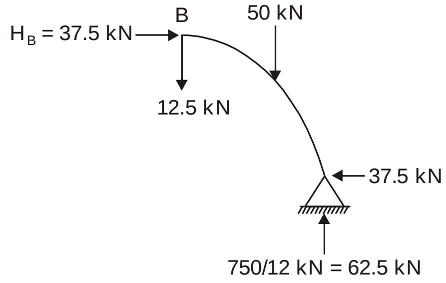
$$R_C = \frac{750}{12} \text{ kN}$$

$BM_{\text{at } B = 0}$ [due to internal hinge]

$$\Rightarrow R_C\times 6 - H\times 6 - 50\times 3 = 0$$

$$\frac{750}{12}\times 6 - 6H - 150 = 0$$

$$H = \frac{375-150}{\sigma} = 37.5 \text{ kN}$$



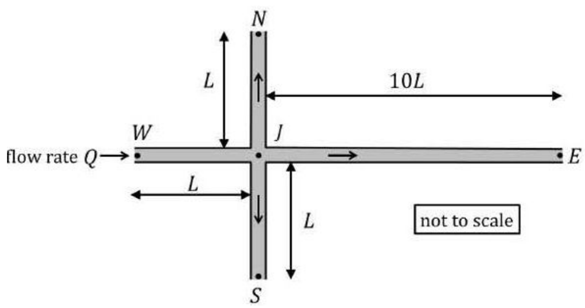

Horizontal reaction at B = 37.5 kN

**54.** In the pipe network shown in the figure, all pipes have the same cross-section and can be assumed to have the same friction factor. The pipes connecting points W, N and S with point J have an equal length L. The pipe connecting points J and E has a length 10L. The pressure at the ends N, E and S are equal. The flow rate in the pipe connecting W and J is Q. Assume that the fluid flow is steady, incompressible, and the pressure losses at the pipe entrance and junction are negligible. Consider the following statements:

I. The flow rate in pipe connecting J and E is Q/21.

II. The pressure difference between J and N is equal to the pressure difference between J and E.

Which one of the following options is CORRECT?



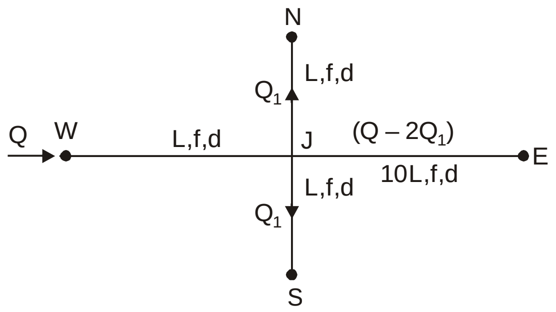

(a) I is False and II is True.

(b) Both I and II are true

(c) Both I and II are False

(d) I is True and II is False

**Sol–54: (a) challenge**




Due to symmetry ($P_N = P_S$) and equal length and friction factor flow distributed in JN and JS will be same. Let it be $Q_1$.

From continuity, flow in JE = $(Q - 2Q_1)$.

$$P_N = P_E = P_S \text{ [Given]}$$

$$P_J - P_N = \frac{flQ_1^2}{12.1d^5} \quad \text{...(A)}$$

$$P_J - P_E = \frac{f(10l)(Q-2Q_1)^2}{12.1d^5} \quad \text{....(B)}$$

From (A) and (B),

$$Q_1^2 = 10(Q - 2Q_1)^2$$

$$\frac{Q-2Q_1}{Q_1} = \frac{1}{\sqrt{10}}$$

$$\frac{Q}{Q_1} = 2+\frac{1}{\sqrt{10}}$$

$$Q_1 = Q\left[\frac{\sqrt{10}Q}{1+2\sqrt{10}}\right]$$

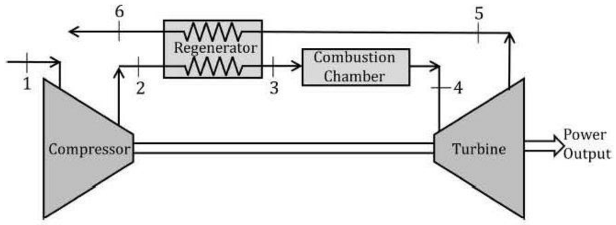
$$Q - Q_1 = Q\left[1-\frac{\sqrt{10}}{1+2\sqrt{10}}\right]$$

$$Q-Q_1 = Q\left[\frac{1+\sqrt{10}}{1+2\sqrt{10}}\right] = \text{Flow in JE}$$

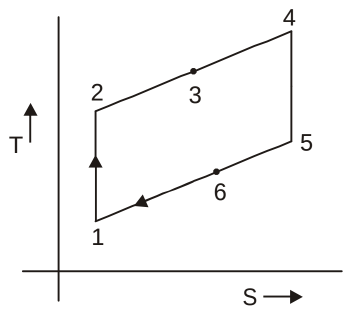
Hence, correct option is (a).

I is False and II is true.

**55.** Consider an air-standard Brayton cycle with adiabatic compressor and turbine, and a regenerator, as shown in the figure. Air enters the compressor at 100 kPa and 300 K and exits the compressor at 600 kPa and 550 K. The air exits the combustion chamber at 1250 K and exits the adiabatic turbine at 100 kPa and 800 K. The exhaust air from the turbine is used to preheat the air in regenerator. The exhaust air exits the regenerator (state 6) at 600 K. There is no pressure drop across the regenerator and the combustion chamber. Also, there is no heat loss from the regenerator to the surroundings. The ratio of specific heats at constant pressure and volume is $c_p/c_v = 1.4$. The thermal efficiency of the cycle is _____ % (answer in integer).




**Sol–55: (40)**




$T_1 = 300$ K

$T_2 = 550$ K

$T_4 = 1250$ K

$T_5 = 800$ K

$T_6 = 600$ K

$\gamma = 1.4$

$W_{net} = C_P (T_4 - T_5) - C_P (T_2 - T_1)$

$Q_R = C_P (T_6 - T_1)$

$Q_S = W_{net} + Q_R$

So, $\eta = \dfrac{W_{net}}{Q_s}$

$$= \frac{C_P(T_4-T_5)-C_P(T_2-T_1)}{C_P(T_4-T_5)-C_P(T_2-T_1)+C_P(T_6-T_1)}$$

$$= \frac{(1250-800)-(550-300)}{(1250-800)-(550-300)+(600-300)}$$

$$= \frac{450-250}{450-250+300} = \frac{200}{500} = 40\%$$