

Sol. $\lim_{t \rightarrow x} \frac{t^{10}f(x) - x^{10}f(t)}{t^9 - x^9} = 1$

$$\lim_{t \rightarrow x} \frac{10t^9f(x) - f'(t)x^{10}}{9t^8} = 1$$

$$\Rightarrow 10x^9f(x) - f'(x)x^{10} = 9x^8$$

$$\Rightarrow f'(x) - \frac{10}{x}f(x) = -\frac{9}{x^2}$$

$$IF = e^{-\int \frac{10}{x} dx} = \frac{1}{x^{10}}$$

\therefore Solⁿ

$$\frac{y}{x^{10}} = \int -\frac{9}{x^{10}} \times \frac{1}{x^2} dx$$

$$= -9 \int x^{-12} dx$$

$$\frac{y}{x^{10}} = \frac{9}{11} x^{-11} + C$$

$$\because y(1) = 2 \Rightarrow C = \frac{13}{11}$$

$$\Rightarrow y = \frac{9}{11x} + \frac{13}{11} x^{10}$$

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is

(A) $\frac{1}{12}$

(B) $\frac{1}{7}$

(C) $\frac{5}{7}$

(D) $\frac{5}{12}$

Answer (C)

Sol. Let $P(\text{knows answer}) = k$

$$P(\text{guesses}) = 1 - k$$

$$P\left(\frac{\text{correct ans}}{\text{guessed}}\right) = \frac{1}{2}$$

$$P\left(\frac{\text{guessed}}{\text{correct answer}}\right) = \frac{P(\text{guessed}) P\left(\frac{\text{correct ans}}{\text{guessed}}\right)}{P(\text{guessed}) P\left(\frac{\text{correct ans}}{\text{guessed}}\right) + P(\text{knows}) P\left(\frac{\text{correct ans}}{\text{knows}}\right)}$$

$$= \frac{(1-k)\left(\frac{1}{2}\right)}{(1-k)\left(\frac{1}{2}\right) + k(1)} = \frac{1}{6}$$

$$\Rightarrow (3-3k) = \frac{1}{2} + \frac{k}{2}$$

$$\Rightarrow \frac{5}{2} = \frac{7k}{2} \Rightarrow k = \frac{5}{7}$$

3. Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$. Then $\left(\sin \frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos \frac{11x}{2}\right)(\sin 6x + \cos 6x)$ is equal to

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$

(B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$

(C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$

(D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

Answer (B)

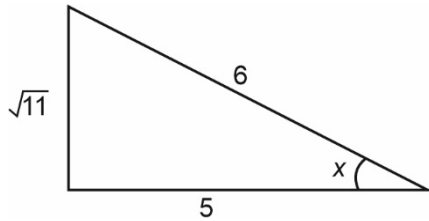
Sol. Let $E = \sin 6x \cos \frac{11x}{2} - \cos 6x \sin \frac{11x}{2} + \cos 6x \cos \frac{11x}{2} + \sin 6x \sin \frac{11x}{2}$

$$E = \sin \frac{x}{2} + \cos \frac{x}{2}$$

Now, $E^2 = 1 + \sin x$

$$\therefore \cot x = \frac{-5}{\sqrt{11}}$$

$$= 1 + \frac{\sqrt{11}}{6}$$



$$\therefore E = \sqrt{\frac{6 + \sqrt{11}}{6}}$$

$$= \sqrt{\frac{12 + 2\sqrt{11}}{12}}$$

$$= \frac{\sqrt{11} + 1}{2\sqrt{3}}$$

4. Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let $S(p, q)$ be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x -coordinate and O be the center of the ellipse. If the area of the triangle $\triangle ORT$ is $\frac{3}{2}$, then which of the following options is correct?

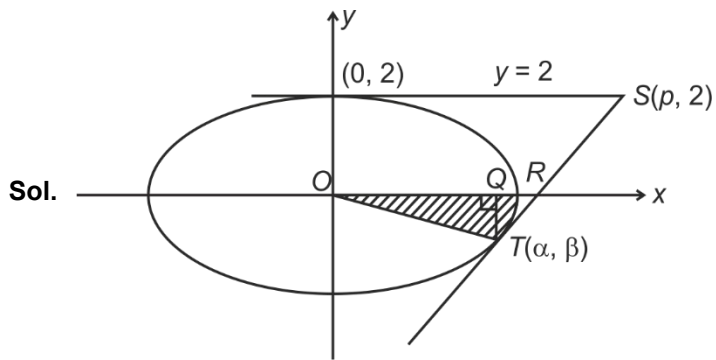
(A) $q = 2, p = 3\sqrt{3}$

(B) $q = 2, p = 4\sqrt{3}$

(C) $q = 1, p = 5\sqrt{3}$

(D) $q = 1, p = 6\sqrt{3}$

Answer (A)



$$q = 2$$

$$\text{Area (ORT)} = \frac{3}{2}$$

$$\Rightarrow \left| \frac{1}{2} \times OR \times QT \right| = \frac{3}{2}$$

$$\Rightarrow \left| \frac{1}{2} \times 3 \times \beta \right| = \frac{3}{2}$$

$$\Rightarrow \beta = -1$$

$$\therefore \frac{\alpha^2}{9} + \frac{\beta^2}{4} = 1$$

$$\Rightarrow \frac{\alpha^2}{9} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \alpha^2 = \frac{27}{4} \Rightarrow \alpha = \frac{3\sqrt{3}}{2}$$

Tangent at T

$$T = 0$$

$$\frac{x \cdot \frac{3\sqrt{3}}{2}}{9} + \frac{y(-1)}{4} = 1 \Big|_{(p, 2)}$$

$$\Rightarrow \frac{p\sqrt{3}}{6} - \frac{1}{2} = 1 \Rightarrow \frac{p\sqrt{3}}{6} = \frac{3}{2} \Rightarrow p = 3\sqrt{3}$$

$$\therefore p = 3\sqrt{3}, q = 2$$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

FULL MARKS : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -2 In all other cases.

5. Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, $T_1 = \{(-1 + \sqrt{2})^n : n \in \mathbb{N}\}$ and $T_2 = \{(1 + \sqrt{2})^n : n \in \mathbb{N}\}$. Then which of the following statements is (are) TRUE?
- (A) $\mathbb{Z} \cup T_1 \cup T_2 \subset S$
- (B) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set
- (C) $T_2 \cap (2024, \infty) \neq \phi$
- (D) For any given $a, b \in \mathbb{Z}$, $\cos(\pi(a + b\sqrt{2})) + i \sin(\pi(a + b\sqrt{2})) \in \mathbb{Z}$ if and only if $b = 0$, where $i = \sqrt{-1}$

Answer (A, C, D)

Sol. $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$

For $b = 0$; $\mathbb{Z} \subset S$

$$T_1 = \{(-1 + \sqrt{2})^n : n \in \mathbb{N}\} \text{ and } T_2 = \{(1 + \sqrt{2})^n : n \in \mathbb{N}\}$$

For $n \in \mathbb{N}$ elements of T_1 and T_2 are of the form $a + b\sqrt{2}$

Hence $Z \cup T_1 \cup T_2 \subset S$

- Now, $-1 + \sqrt{2} < 1$ and its higher powers decreases

$\Rightarrow (-1 + \sqrt{2})^n < 1$ and can be made in $\left(0, \frac{1}{2024}\right)$ for some higher n .

- $1 + \sqrt{2} > 1$ and its higher power increases

$\Rightarrow (1 + \sqrt{2})^n$ can be made in $(2024, \infty)$ for some higher n .

- $\cos \pi(a + b\sqrt{2}) + i \sin \pi(a + b\sqrt{2}) \in Z$ if

$a + b\sqrt{2}$ is an integer $\Rightarrow b = 0$

6. Let \mathbb{R}^2 denote $\mathbb{R} \times \mathbb{R}$. Let

$S = \{(a, b, c) : a, b, c \in \mathbb{R} \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x, y) \in \mathbb{R}^2 - \{(0, 0)\}\}$.

Then which of the following statements is (are) TRUE?

(A) $\left(2, \frac{7}{2}, 6\right) \in S$

(B) If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$

(C) For any given $(a, b, c) \in S$, the system of linear equations $\begin{cases} ax + by = 1 \\ bx + cy = -1 \end{cases}$ has a unique solution.

(D) For any given $(a, b, c) \in S$, the system of linear equations $\begin{cases} (a+1)x + by = 0 \\ bx + (c+1)y = 0 \end{cases}$ has a unique solution.

Answer (B, C, D)

Sol. $ax^2 + 2bxy + cy^2 > 0$

$y, x \in \mathbb{R} - \{(0, 0)\}$

$$\Rightarrow c\left(\frac{y}{x}\right)^2 + 2b\left(\frac{y}{x}\right) + a > 0$$

$$\Rightarrow c > 0, D < 0$$

$$4b^2 - 4ac < 0$$

$$\Rightarrow b^2 < ac$$

$$(A) \left(2, \frac{7}{2}, 6\right)$$

$$\left(\frac{7}{2}\right)^2 > 2 \times 6$$

∴ option A is incorrect

$$(B) \text{ If } \left(3, b, \frac{1}{12}\right) \in S$$

$$\Rightarrow b^2 < 3 \cdot \frac{1}{12}$$

$$\Rightarrow b^2 < \frac{1}{4}$$

$$\Rightarrow 4b^2 < 1$$

$$\Rightarrow |2b| < 1 \text{ option B is correct}$$

$$(C) ax + by = 1$$

$$bx + cy = -1$$

$$D = \begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 \neq 0$$

∴ unique solution option C is correct.

$$(D) (a + 1)x + by = 0$$

$$bx + (c + 1)y = 0$$

$$D = \begin{vmatrix} (a+1) & b \\ b & (c+1) \end{vmatrix}$$

$$= (a + 1)(c + 1) - b^2$$

$$\Rightarrow ac - b^2 + a + c + 1$$

$$b^2 < ac \Rightarrow ac \text{ is +ve}$$

$$\Rightarrow a \text{ and } c \text{ are positive then } (ac - b^2) + a + c + 1 > 0 \therefore \text{ unique solution}$$

∴ option D is correct.

7. Let \mathbb{R}^3 denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$. Let $\text{dist}(X, Y)$ denote the distance between two points X and Y in \mathbb{R}^3 . Let

$$S = \{X \in \mathbb{R}^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\} \text{ and}$$

$$T = \{Y \in \mathbb{R}^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\}.$$

Then which of the following statements is (are) TRUE?

- (A) There is a triangle whose area is 1 and all of whose vertices are from S .
- (B) There are two distinct points L and M in T such that each point on the line segment LM is also in T .
- (C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T .
- (D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T .

Answer (A, B, C)

Sol. $S : \{(x-1)^2 + (y-2)^2 + (z-3)^2 - ((x-4)^2 + (y-2)^2 + (z-7)^2) = 50\}$

$\Rightarrow S : \{6x + 8z - 105 = 0\}$

Similarly $T = \{6x + 8z - 5 = 0\}$

S represents a plane. So it will contain a triangle of area 1. So (A) is correct.

T represents a plane. So (B) is correct.

S & T are two parallel planes at a distance of 10 units from each other.

\therefore (C) is correct and (D) is incorrect.

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

8. Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{\frac{1}{6}}\sqrt{6}}$. If $x, y \in \mathbb{R}$ are such that

$3x + 2y = \log_a(18)^{\frac{5}{4}}$ and

$2x - y = \log_b(\sqrt{1080})$,

then $4x + 5y$ is equal to _____.

Answer (8)

Sol. $a = 3\sqrt{2} \Rightarrow a^2 = 18$

Notice that $1080 = 5 \cdot 6^3 \Rightarrow$

$5^{\frac{1}{6}} \cdot 6^{\frac{1}{2}} = (1080)^{\frac{1}{6}} = \frac{1}{b} \Rightarrow 1080^{\frac{1}{2}} = \frac{1}{b^3}$

$\Rightarrow 3x + 2y = \log_a(a^2)^{\frac{5}{4}} = \frac{5}{2} \dots(i)$

$$2x - y = \log_b \frac{1}{b^3} = \log_b b^{-3} = -3 \quad \dots(ii)$$

⇒ Solving (i) & (ii)

$$\Rightarrow x = \frac{-1}{2}, y = 2 \Rightarrow 4x + 5y = 8$$

9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to _____.

Answer (20)

$$\text{Sol. } \because f(1) = -9 \Rightarrow 1 + a + b + c = -9 \quad \dots(1)$$

$$4x^3 + 3ax^2 + 2bx = 0$$

$$\Rightarrow x = 0, \quad 4x^2 + 3ax + 2b = 0 \quad \dots(2)$$

⇒ $\sqrt{3}i$ and $-\sqrt{3}i$ are roots of (2)

$$\Rightarrow \sqrt{3}i - \sqrt{3}i = \frac{-3a}{4}, \quad \sqrt{3}i(-\sqrt{3}i) = \frac{2b}{4}$$

$$\Rightarrow a = 0, b = 6, c = -16$$

$$\Rightarrow f(x) = 0 \Rightarrow x^4 + 6x^2 - 16 = 0$$

$$\Rightarrow x^2 = \frac{-6 \pm \sqrt{36 + 64}}{2} = -3 \pm 5 = 2, -8$$

$$x = -\sqrt{2}, +\sqrt{2}, -2\sqrt{2}i, 2\sqrt{2}i$$

$$\Rightarrow |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$$

10. Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$, where $|A|$ denotes the determinant of A . Then the

number of elements in S is _____.

Answer (16)

$$\text{Sol. } |A| = -(e - d) + c(b - a) = \pm 1$$

Case (i) : $c = 0 \Rightarrow (e, d) = (1, 0), (0, 1) \rightarrow 2$ ways

b and a can be each 2 ways

⇒ Total = 8 ways

Case (ii) : $c \neq 0 \Rightarrow c = 1$

$$\Rightarrow d - e + b - a = \pm 1$$

$$\left. \begin{array}{l} 1 \ 1 \ 1 \ 0 \\ 1 \ 1 \ 0 \ 1 \\ 0 \ 0 \ 1 \ 0 \\ 0 \ 0 \ 0 \ 1 \end{array} \right\} \rightarrow 4 \times 2 = 8 \text{ ways}$$

Total = 16 ways

11. A group of 9 students, s_1, s_2, \dots, s_9 , is to be divided to form three teams X, Y and, Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is_____.

Answer (665)

Sol. Number of required ways

$$\begin{aligned} &= \frac{9!}{2!3!4!} - (n(s_1 \in X) + n(s_2 \in Y) - n(s_1 \in X \text{ and } s_2 \in Y)) \\ &= \frac{9!}{2!3!4!} - \left(\frac{8!}{1!3!4!} + \frac{8!}{2!2!4!} - \frac{7!}{1!2!4!} \right) \\ &= 665 \end{aligned}$$

12. Let $\overline{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overline{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overline{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $(\overline{OP} \times \overline{OQ}) \cdot \overline{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of l is _____.

Answer (5)

Sol. $(\overline{OP} \times \overline{OQ}) \cdot \overline{OR} = 0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0 \Rightarrow \frac{\alpha-1}{\alpha} \left(\frac{\beta-1}{2\beta} - 1 \right) - \left(\frac{1}{2} - 1 \right) + 1 \left(1 - \frac{\beta-1}{\beta} \right) = 0$$

$$\frac{\alpha-1}{\alpha} \left(\frac{-\beta-1}{2\beta} \right) + \frac{1}{2} + \frac{1}{\beta} = 0$$

$$\Rightarrow \frac{\beta+2}{2\beta} = \frac{\alpha\beta + \alpha - \beta - 1}{2\alpha\beta}$$

$$\Rightarrow \alpha\beta + 2\alpha = \alpha\beta + \alpha - \beta - 1 \Rightarrow \boxed{\alpha + \beta + 1 = 0} \quad \dots (1)$$

Now $(\alpha, \beta, 2)$ lies on $3x + 3y - z + l = 0$

$$\Rightarrow 3(\alpha + \beta) - 2 + l = 0 \quad \dots (2)$$

$$\Rightarrow -3 - 2 + l = 0 \Rightarrow l = 5$$

13. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x . Suppose that the points $(x, P(X = x))$, $x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy -plane, and $P(X = x) = 0$ for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α , then the value of 24α is _____.

Answer (42)

$$\text{Sol. } \sum_{x=0}^4 xP(x) = \frac{5}{2}$$

$$\sum_{x=0}^4 x^2P(x) = ?$$

$(0, P(0)), (1, P(1)), (2, P(2)), (3, P(3)), (4, P(4))$

$$K = P(1) - P(0) = P(2) - P(1) = P(3) - P(2) = P(4) - P(3)$$

$$P(1) = K + P(0)$$

$$P(2) = 2K + P(0)$$

$$P(3) = 3K + P(0)$$

$$P(4) = 4K + P(0)$$

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$\Rightarrow 5P(0) + 10K = 1$$

$$K + P(0) + 4K + 2P(0) + 9K + 3P(0) + 16K + 4P(0) = \frac{5}{2}$$

$$30K + 10P(0) = \frac{5}{2}$$

$$\therefore 10K = \frac{1}{2}$$

$$K = \frac{1}{20}, P(0) = \frac{1}{10}$$

$$P(1) = \frac{3}{20}, P(2) = \frac{4}{20}, P(3) = \frac{5}{20}, P(4) = \frac{6}{20}$$

$$\sum_{x=0}^4 x^2P(x) = 8$$

$$\therefore \text{Variance} = 8 - \frac{25}{4} = \frac{32 - 25}{4} = \frac{7}{4}$$

$$\therefore 24\alpha = \frac{24 \times 7}{4} = 42$$

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

- 14.** Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3 \times 3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$. Match each entry in **List-I** to the correct entry in **List-II**.

	List-I		List-II
(P)	The number of matrices $M = (a_{ij})_{3 \times 3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3 \times 3}$ with all entries in T such that $C_j = 0$ for all j is	(2)	12
(R)	Let $M = (a_{ij})_{3 \times 3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in R, M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix} \right\}$ is	(3)	Infinite
(S)	Let $M = (a_{ij})_{3 \times 3}$ be a matrix with all entries in T such that $R_i = 0$ for all i . Then the absolute value of the determinant of M is	(4)	6
		(5)	0

The correct option is

- (A) (P) → (4) (Q) → (2) (R) → (5) (S) → (1)
 (C) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

- (B) (P) → (2) (Q) → (4) (R) → (1) (S) → (5)
 (D) (P) → (1) (Q) → (5) (R) → (3) (S) → (4)

Answer (C)

Sol. $x^2 + x - 1 = 0 \rightarrow$ roots are α and β

$$\alpha + \beta = -1 \quad \alpha\beta = -1$$

Set $T = \{1, \alpha, \beta\}$ $M = (a_{ij})_{3 \times 3}$

$$R_i = a_{i1} + a_{i2} + a_{i3} \quad C_j = a_{1j} + a_{2j} + a_{3j}$$

(P) $R_i = C_j = 0$ for all i, j

$$\alpha + \beta = -1 \quad T = \{1, \alpha, \beta\}$$

$$\begin{aligned} \text{Number of matrices} \\ = \underline{3} \times 2 \times 1 = 12 \end{aligned}$$

↓
 Number of ways to arrange 1, α , β in R_1 Number of ways to arrange 1, α , β in R_2

$$\begin{bmatrix} 1 & \alpha & \beta \\ - & - & - \end{bmatrix}$$

(Q) Number of symmetric matrices = ?

$$C_j = 0 \quad \forall j$$

Number of symmetric matrices

$$= \underline{3} \times 1 = 6 \quad \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

(R) $M \rightarrow$ skew symmetric of 3×3

$$|M| = 0 \quad a_{ij} \in T \text{ for } i > j$$

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}$$

$$\begin{bmatrix} 0 & -a_{21} & -a_{31} \\ a_{21} & 0 & -a_{32} \\ a_{31} & a_{32} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{12} \\ 0 \\ -a_{23} \end{bmatrix}$$

As $x, y, z \in R$ and a_{12} & $a_{23} \in R$
 & $|M| = 0$

\therefore System has infinite solutions

(S) $R_i = 0 \forall i$

$$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3 \quad |M| = \begin{vmatrix} 1+\alpha+\beta & \alpha & \beta \\ 1+\alpha+\beta & \beta & 1 \\ 1+\alpha+\beta & 1 & \alpha \end{vmatrix} = 0$$

(P) \rightarrow (2) (Q) \rightarrow (4) (R) \rightarrow (3) (S) \rightarrow (5)

15. Let the straight line $y = 2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1 . Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

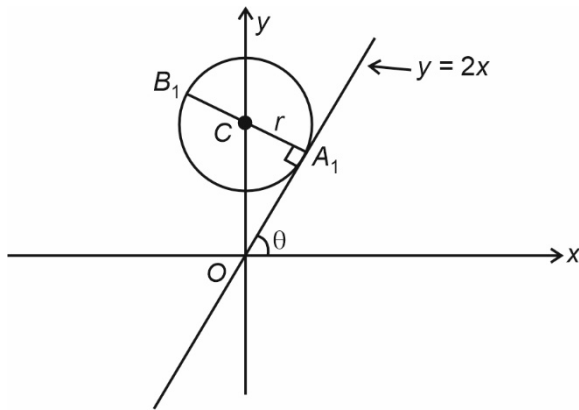
	List-I		List-II
(P)	α equals	(1)	$(-2, 4)$
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	$(-2, 6)$
(S)	B_1 equals	(4)	5
		(5)	$(2, 4)$

The correct option is

- (A) (P) \rightarrow (4) (Q) \rightarrow (2) (R) \rightarrow (1) (S) \rightarrow (3) (B) (P) \rightarrow (2) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (3)
 (C) (P) \rightarrow (4) (Q) \rightarrow (2) (R) \rightarrow (5) (S) \rightarrow (3) (D) (P) \rightarrow (2) (Q) \rightarrow (4) (R) \rightarrow (3) (S) \rightarrow (5)

Answer (C)

Sol.



Slope of line = 2 $\Rightarrow \tan\theta = 2$

$C(0, \alpha) \quad \alpha > 0$

$$\alpha + r = 5 + \sqrt{5} \quad \dots(1)$$

Line $y = 2x$ is tangent to the circle

$$\therefore \left| \frac{0 - \alpha}{\sqrt{4 + 1}} \right| = r$$

$$\Rightarrow |-\alpha| = r\sqrt{5}$$

$$\Rightarrow \alpha = r\sqrt{5} \quad \text{as } \alpha > 0$$

From equation (1) $r\sqrt{5} + r = 5 + \sqrt{5}$

$$\Rightarrow r(\sqrt{5} + 1) = \sqrt{5}(\sqrt{5} + 1)$$

$$\Rightarrow r = \sqrt{5}$$

And $\alpha = r\sqrt{5} = \sqrt{5} \times \sqrt{5} = 5$

Centre $C(0, 5)$

$$OC = 5$$

$$A_1C = \sqrt{5}$$

$$\therefore OA_1 = \sqrt{25 - 5} = \sqrt{20} = 2\sqrt{5}$$

$$\tan\theta = 2$$

(from figure)

$$\cos\theta = \frac{1}{\sqrt{5}} \quad \sin\theta = \frac{2}{\sqrt{5}}$$

$$A_1(0 + OA_1 \cos\theta, 0 + OA_1 \sin\theta)$$

$$A_1\left(2\sqrt{5} \times \frac{1}{\sqrt{5}}, 2\sqrt{5} \times \frac{2}{\sqrt{5}}\right)$$

$$A_1(2, 4)$$

Let $B_1(x_1, y_1)$

$$\therefore \frac{x_1 + 2}{2} = 0 \quad \text{and} \quad \frac{y_1 + 4}{2} = 5$$

$$x_1 = -2 \quad y_1 = 6$$

$$B_1(-2, 6)$$

$$\alpha = 5 \quad r = \sqrt{5} \quad A_1(2, 4) \quad B_1(-2, 6)$$

16. Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2 . Let $O = (0, 0, 0)$, and \hat{n} denote a unit normal vector to the plane containing both the lines L_1 and L_2 .

Match each entry in **List-I** to the correct entry in **List-II**.

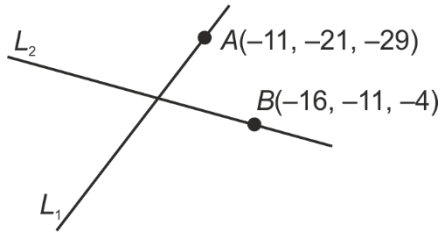
	List-I		List-II
(P)	γ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$
(Q)	A possible choice for \hat{n} is	(2)	$\sqrt{\frac{3}{2}}$
(R)	$\overline{OR_1}$ equals	(3)	1
(S)	A possible value of $\overline{OR_1} \cdot \hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is

- (A) (P) \rightarrow (3) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (2)
 (B) (P) \rightarrow (5) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (2)
 (C) (P) \rightarrow (3) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (5)
 (D) (P) \rightarrow (3) (Q) \rightarrow (1) (R) \rightarrow (4) (S) \rightarrow (5)

Answer (C)

Sol. Vector parallel to the line L_1 (say \vec{b}_1) = $\hat{i} + 2\hat{j} + 3\hat{k}$



Normal vector of plane (\vec{n}) containing L_1 and L_2 will be perpendicular to both \vec{b}_1 and \vec{AB}

$$\Rightarrow \vec{n} = p(\vec{AB} \times \vec{b}_1) = p(5\hat{i} - 10\hat{j} - 25\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= p(20\hat{i} - 40\hat{j} + 20\hat{k})$$

$$\Rightarrow \hat{n} = \frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

Now, vector parallel to L_2 (say \vec{b}_2) is perpendicular to $\vec{n} \Rightarrow \vec{b}_2 \cdot \vec{n} = 0$

$$(3\hat{i} + 2\hat{j} + \gamma\hat{k}) \cdot p(20\hat{i} - 40\hat{j} + 20\hat{k}) = 0$$

$$\Rightarrow \gamma = 1$$

Now, for point of intersection (POI)

$$L_1: \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = \lambda \text{ and } L_2: \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma} = u$$

Comparing x and y coordinates, $-11 + \lambda = -16 + 3u$ and $-21 + 2\lambda = -11 + 2u$

$$\Rightarrow \lambda = 10, u = 5$$

$$\Rightarrow \text{POI i.e., } \overline{OR_1}: (-\hat{i} - \hat{j} + \hat{k}) \text{ and } \overline{OR} \cdot \hat{n} = \sqrt{\frac{2}{3}}$$

17. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x |x| \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \text{ and } g(x) = \begin{cases} 1-2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}$$

Let $a, b, c, d \in \mathbb{R}$. Define the function $h: \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = af(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c(x - g(x)) + d g(x), x \in \mathbb{R}$$

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		List-II	
(P)	If $a = 0, b = 1, c = 0$ and $d = 0$, then	(1)	h is one-one
(Q)	If $a = 1, b = 0, c = 0$ and $d = 0$, then	(2)	h is onto.
(R)	If $a = 0, b = 0, c = 1$ and $d = 0$, then	(3)	h is differentiable on \mathbb{R} .
(S)	If $a = 0, b = 0, c = 0$ and $d = 1$, then	(4)	the range of h is $[0, 1]$
		(5)	the range of h is $\{0, 1\}$

The correct option is

(A) (P) \rightarrow (4) (Q) \rightarrow (3) (R) \rightarrow (1) (S) \rightarrow (2)

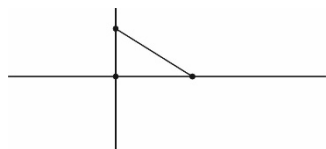
(B) (P) \rightarrow (5) (Q) \rightarrow (2) (R) \rightarrow (4) (S) \rightarrow (3)

(C) (P) \rightarrow (5) (Q) \rightarrow (3) (R) \rightarrow (2) (S) \rightarrow (4)

(D) (P) \rightarrow (4) (Q) \rightarrow (2) (R) \rightarrow (1) (S) \rightarrow (3)

Answer (C)

Sol. $g(x) = \begin{cases} 1-2x, & 0 \leq x \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$



$$g\left(\frac{1}{2}-x\right) = \begin{cases} 1-2\left(\frac{1}{2}-x\right), & 0 \leq \frac{1}{2}-x \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$g(x) + g\left(\frac{1}{2}-x\right) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Now, option (P), at $b = 1$

$$h(x) = g(x) + g\left(\frac{1}{2}-x\right) \text{ has range } \{0, 1\}$$

(P) \rightarrow (5)

Option (Q) at $a = 1$, $h(x) = f(x)$

$$f'(0^+) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \sin \frac{1}{h}}{h}$$

$$f'(0^-) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} h \sin h = 0$$

$$\Rightarrow f'(0^+) = f'(0^-)$$

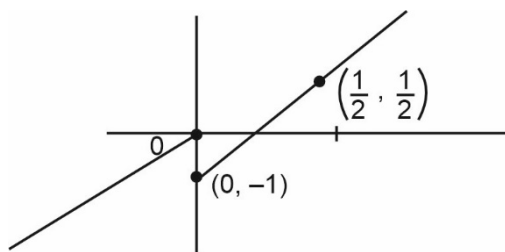
as $f(0^+) = f(0^-)$

$\Rightarrow h(x) = f(x)$ is differentiable at $x = 0$ and all other points $f(x) = h(x)$ is differentiable as product of two differentiable functions

(Q) \rightarrow (3)

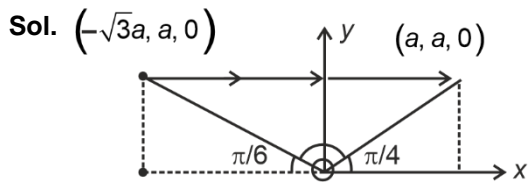
Option (R)

$$h(x) = x - g(x) = \begin{cases} 3x - 1, & 0 \leq x \leq \frac{1}{2} \\ x, & \text{otherwise} \end{cases}$$



by graph $h(x)$ has range \mathbb{R} and onto (R) \rightarrow (2)

(S) at $d = 1$ $h(x) = g(x)$ has range $[0, 1]$ (S) \rightarrow (4)



$$\theta = \pi - \frac{\pi}{4} - \frac{\pi}{6}$$

$$\Rightarrow \theta = \frac{12\pi - 3\pi - 2\pi}{12} = \frac{7\pi}{12}$$

So, $\int \vec{B} \cdot d\vec{l}$ along the line is

$$\int \vec{B} \cdot d\vec{l} = -\frac{\mu_0(I)}{2\pi} \cdot \theta = \frac{\mu_0 I}{2\pi} \cdot \frac{7\pi}{12}$$

$$\Rightarrow \left| \int \vec{B} \cdot d\vec{l} \right| = \frac{7\mu_0 I}{24}$$

Option (A) is correct Answer.

3. Two beads, each with charge q and mass m , are on a horizontal, frictionless, non-conducting, circular hoop of radius R . One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by

[ϵ_0 is the permittivity of free space.]

(A) $q^2 / (4\pi\epsilon_0 R^3 m)$

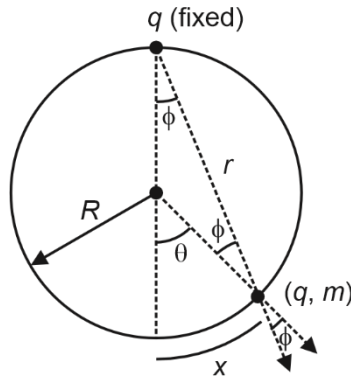
(B) $q^2 / (32\pi\epsilon_0 R^3 m)$

(C) $q^2 / (8\pi\epsilon_0 R^3 m)$

(D) $q^2 / (16\pi\epsilon_0 R^3 m)$

Answer (B)

Sol. As the hoop mass is not given so it must not move or else its inertia must have some effect.



	List-I		List-II
(P)	If $n = 2$ and $\alpha = 180^\circ$, then all the possible values of θ_0 will be	(1)	30° and 0°
(Q)	If $n = \sqrt{3}$ and $\alpha = 180^\circ$, then all the possible values of θ_0 will be	(2)	60° and 0°
(R)	If $n = \sqrt{3}$ and $\alpha = 180^\circ$, then all the possible values of ϕ_0 will be	(3)	45° and 0°
(S)	If $n = \sqrt{2}$ and $\theta_0 = 45^\circ$, then all the possible values of α will be	(4)	150°
		(5)	0°

(A) $P \rightarrow 5; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$

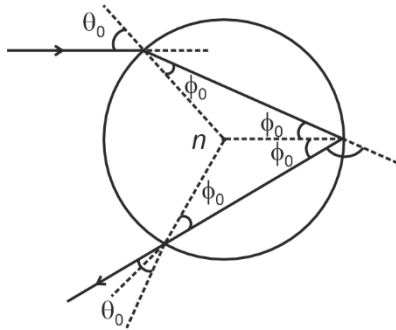
(B) $P \rightarrow 5; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 4$

(C) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$

(D) $P \rightarrow 3; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 5$

Answer (A)

Sol.



$$\alpha = (\theta_0 - \phi_0) + (\pi - 2\phi_0) + (\theta_0 - \phi_0)$$

$$\alpha = \pi + 2\theta_0 - 4\phi_0 \quad \dots(i)$$

$$\sin \theta_0 = n \sin \phi_0 \quad \dots(ii)$$

For (P)

$$n = 2, \alpha = 180$$

$$\text{if } \alpha = \pi, 2\theta_0 - 4\phi_0 = 0$$

$$\theta_0 = 2\phi_0$$

$$\sin \theta_0 = 2 \sin \left(\frac{\theta_0}{2} \right)$$

P → (5)

For Q, $n = \sqrt{3}$, $\alpha = 180$

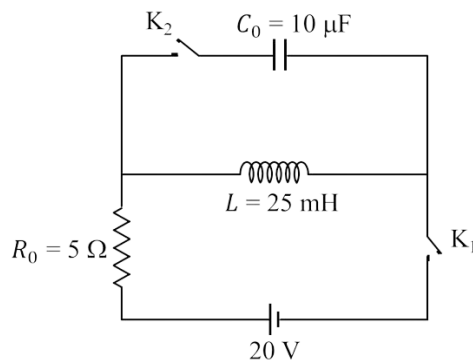
$$\theta_0 = 2\phi_0$$

Q → (2)

$$\sin \theta_0 = \sqrt{3} \sin \left(\frac{\theta_0}{2} \right)$$

$$\theta_0 = 60, 0^\circ$$

17. The circuit shown in the figure contains an inductor L , a capacitor C_0 , a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2 . Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1 . After a long time, the current attains a steady state value I_2 . Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0 .



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The value of I_1 in Ampere is	(1)	0
(Q)	The value of I_2 in Ampere is	(2)	2
(R)	The value of ω_0 in kilo-radians/s is	(3)	4
(S)	The value of V_0 in Volt is	(4)	20
		(5)	200

(A) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 5$

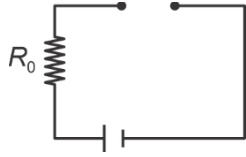
(C) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 4$

(B) $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 5$

(D) $P \rightarrow 2; Q \rightarrow 5; R \rightarrow 3; S \rightarrow 4$

Answer (A)

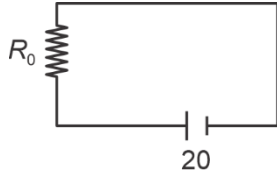
Sol. Just after closing K_1



$i_1 = 0,$

$P \rightarrow (1)$

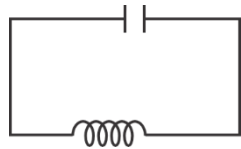
After long time



$i_2 = \frac{20}{5} = 4A,$

$Q \rightarrow (3)$

After opening K_1 & closing K_2



$\omega = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-6}}}$

$\omega = 2000 \text{ rad/s}$

$\omega = 2 \text{ krad/s}$

$R \rightarrow (2)$

$i_0 = 4$

$Q_0 = \frac{i_0}{\omega} = \frac{4}{2 \times 10^3} = 2 \text{ mC}$

$\therefore V_0 = \frac{Q_0}{C} = \frac{2 \times 10^3}{10} = 200$

$S \rightarrow (5)$

Volume and temperature remains same.

$$P_X V = \frac{W_X}{M_X} RT$$

$$P_Y V = \frac{W_Y}{M_Y} RT$$

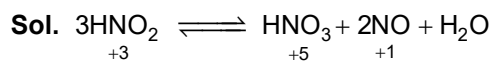
$$M_X \propto \frac{W_X}{P_X}$$

$$M_Y \propto \frac{W_Y}{P_Y}$$

$$\frac{(V_{rms})_X}{(V_{rms})_Y} = \sqrt{\frac{W_Y \cdot P_X}{P_Y \cdot W_X}} = \sqrt{\frac{80}{4} \times \frac{2}{10}} = \sqrt{4} = \frac{2}{1} = 2:1$$

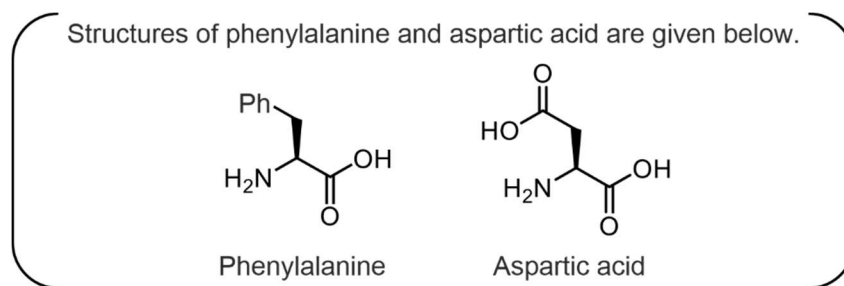
2. At room temperature, disproportionation of an aqueous solution of *in situ* generated nitrous acid (HNO_2) gives the species
- (A) H_3O^+ , NO_3^- and NO
- (B) H_3O^+ , NO_3^- and NO_2
- (C) H_3O^+ , NO^- and NO_2
- (D) H_3O^+ , NO_3^- and N_2O

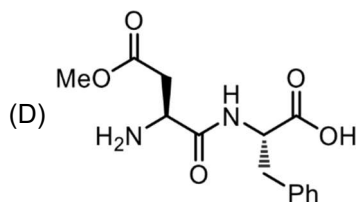
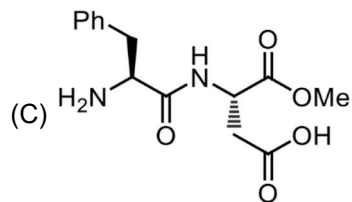
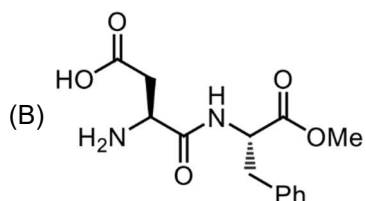
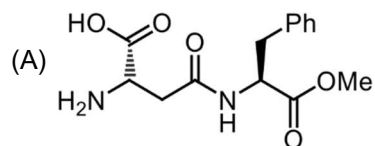
Answer (A)



H_3O^+ , NO_3^- and NO

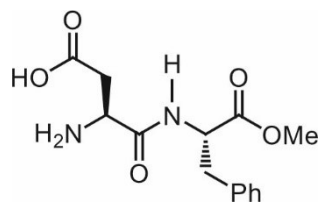
3. Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is





Answer (B)

Sol. Aspartame is



4. Among the following options, select the option in which each complex in **Set-I** shows geometrical isomerism and the two complexes in **Set-II** are ionization isomers of each other.

[en = H₂NCH₂CH₂NH₂]

(A) **Set-I:** [Ni(CO)₄] and [PdCl₂(PPh₃)₂]

Set-II: [Co(NH₃)₅Cl]SO₄ and [Co(NH₃)₅(SO₄)]Cl

(B) **Set-I:** [Co(en)(NH₃)₂Cl₂] and [PdCl₂(PPh₃)₂]

Set-II: [Co(NH₃)₆][Cr(CN)₆] and [Cr(NH₃)₆][Co(CN)₆]

(C) **Set-I:** [Co(NH₃)₃(NO₂)₃] and [Co(en)₂Cl₂]

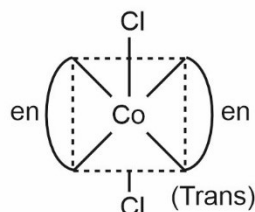
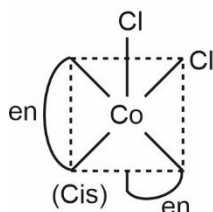
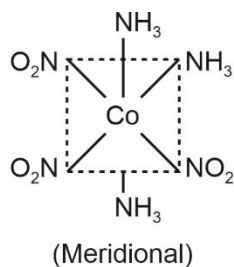
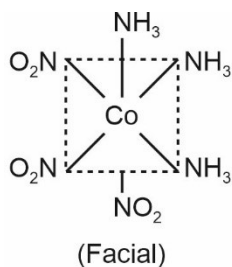
Set-II: [Co(NH₃)₅Cl]SO₄ and [Co(NH₃)₅(SO₄)]Cl

(D) **Set-I:** [Cr(NH₃)₅Cl]Cl₂ and [Co(en)(NH₃)₂Cl₂]

Set-II: [Cr(H₂O)₆]Cl₃ and [Cr(H₂O)₅Cl]Cl₂·H₂O

Answer (C)

Sol. Set-I:



Set-II: $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{SO}_4$ and $[\text{Co}(\text{NH}_3)_5\text{SO}_4]\text{Cl}$ are ionisation isomers.

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -2 In all other cases.

-
5. Among the following the correct statement(s) for electrons in an atom is(are)
- (A) Uncertainty principle rules out the existence of definite paths for electrons.
- (B) The energy of an electron in 2s orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.
- (C) According to Bohr's model, the most negative energy value for an electron is given by $n = 1$, which corresponds to the most stable orbit.
- (D) According to Bohr's model, the magnitude of velocity of electrons increases with increase in values of n .

Answer (A, B, C)

Sol. (A) Uncertainty principle rules out existence of definite paths or trajectories of electron and other similar particles.

So, option (A) is correct.

(B) Shell or orbit more near to nucleus has less energy than faraway.

So, option (B) is also correct.

$$(C) E = -13.6 \frac{Z^2}{n^2} \text{ eV/atom}$$

So, $n = 1$ has most negative energy.

So, option (C) is also correct.

$$(D) V = V_0 \times \frac{Z}{n}$$

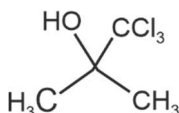
when n increases velocity decreases.

So, option (D) is incorrect.

6. Reaction of *iso*-propylbenzene with O_2 followed by the treatment with H_3O^+ forms phenol and a by-product **P**. Reaction of **P** with 3 equivalents of Cl_2 gives compound **Q**. Treatment of **Q** with $Ca(OH)_2$ produces compound **R** and calcium salt **S**.

The correct statement(s) regarding **P**, **Q**, **R** and **S** is(are)

(A) Reaction of **P** with **R** in the presence of KOH followed by acidification gives

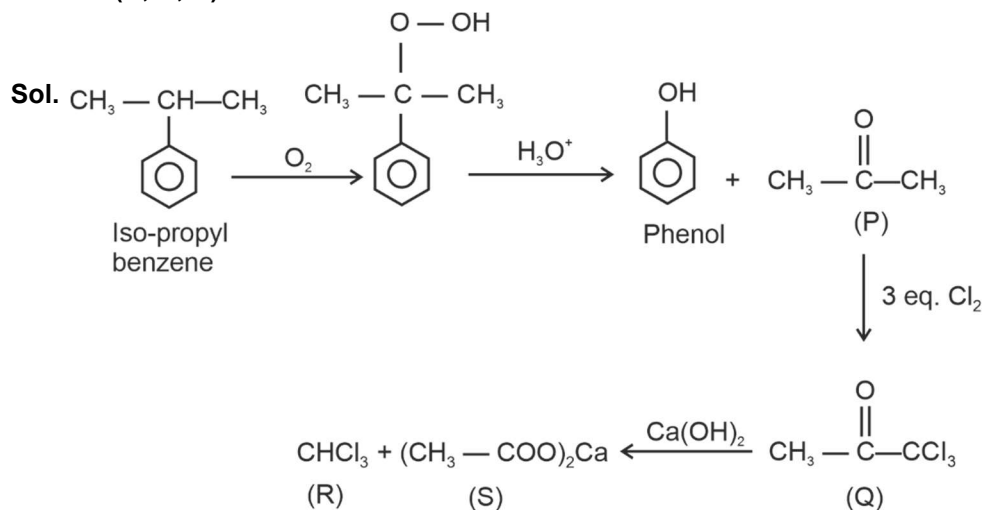


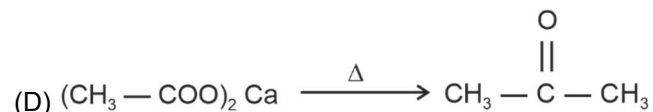
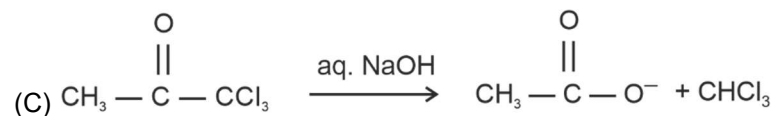
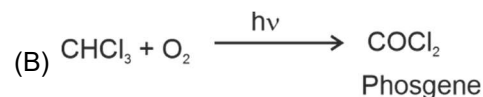
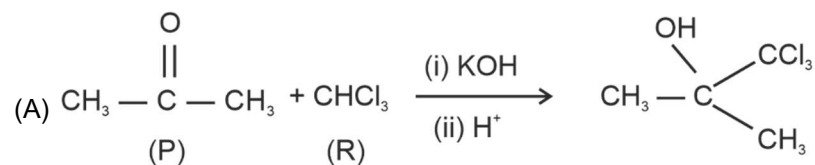
(B) Reaction of **R** with O_2 in the presence of light gives phosgene gas

(C) **Q** reacts with aqueous NaOH to produce Cl_3CCH_2OH and $Cl_3CCOONa$

(D) **S** on heating gives **P**

Answer (A, B, D)





7. The option(s) in which at least three molecules follow Octet Rule is(are)

(A) CO₂, C₂H₄, NO and HCl

(B) NO₂, O₃, HCl and H₂SO₄

(C) BCl₃, NO, NO₂ and H₂SO₄

(D) CO₂, BCl₃, O₃ and C₂H₄

Answer (A, D)

Sol. (A) CO₂, C₂H₄ and HCl follow octet rule.

(B) O₃ and HCl and follow octet rule.

(C) None of them follow octet rule.

(D) CO₂, O₃ and C₂H₄ follow octet rule.

Correct answer is (A) and (D)

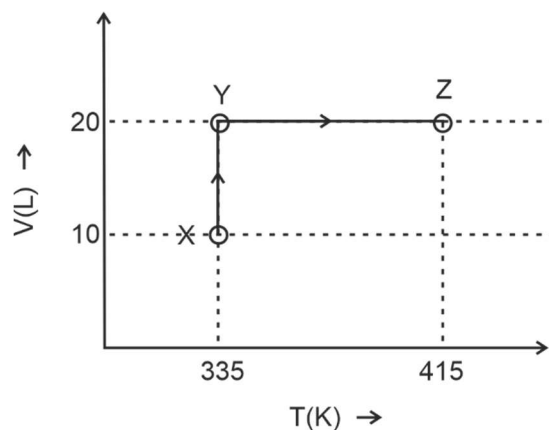
SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

8. Consider the following volume–temperature (V – T) diagram for the expansion of 5 moles of an ideal monoatomic gas.



Considering only P-V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence $X \rightarrow Y \rightarrow Z$ is _____.

[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m} = 12 \text{ J K}^{-1} \text{ mol}^{-1}$ and gas constant, $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$]

Answer (8120)

Sol. $X \rightarrow Y$ is an isothermal process an ideal gas:

$$\Delta H = 0$$

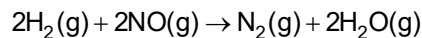
$Y \rightarrow Z$ is an isochoric process

$$\therefore w = 0$$

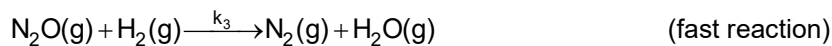
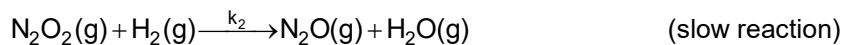
$$\begin{aligned} \Delta U &= nC_{V,m} (T_2 - T_1) \\ &= 5 \times 12 (415 - 335) \\ &= 4800 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta H &= \Delta U + \Delta(PV) \\ &= \Delta U + nR\Delta T \\ &= 4800 + 5 \times 8.3 \times (415 - 335) \\ &= 8120 \text{ J} \end{aligned}$$

9. Consider the following reaction,



which follows the mechanism given below:



The order of the reaction is _____ ?

Answer (3)

Sol. Rate of reaction (according to slowest step)

$$\Rightarrow r = k_2[\text{N}_2\text{O}_2][\text{H}_2] \quad \dots(1)$$

Now for intermediate $[\text{N}_2\text{O}_2]$,

$$\frac{k_1}{k_{-1}} = \frac{[\text{N}_2\text{O}_2]}{[\text{NO}]^2}$$

$$\Rightarrow [\text{N}_2\text{O}_2] = \frac{k_1}{k_{-1}}[\text{NO}]^2 \quad \dots(2)$$

from equation (1) and (2)

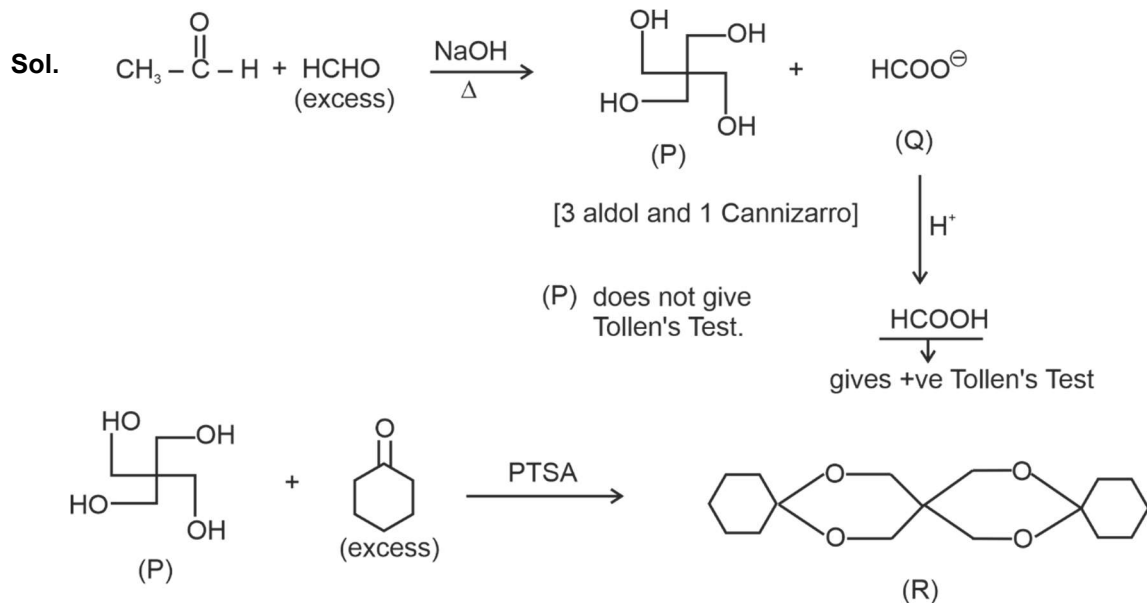
$$r = \frac{k_2 k_1}{k_{-1}} [\text{NO}]^2 [\text{H}_2]$$

Overall order of reaction = 2 + 1 = 3

10. Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives **P** and **Q**. Compound **P** does not give Tollens' test, whereas **Q** on acidification gives positive Tollens' test. Treatment of **P** with excess cyclohexanone in the presence of catalytic amount of p-toluenesulfonic acid (PTSA) gives product **R**.

Sum of the number of methylene groups ($-\text{CH}_2-$) and oxygen atoms in **R** is _____.

Answer (18)



Number of CH_2 groups in R = 14

Number of O-atoms = 4

Required Answer = 14 + 4 = 18

11. Among $\text{V}(\text{CO})_6$, $\text{Cr}(\text{CO})_5$, $\text{Cu}(\text{CO})_3$, $\text{Mn}(\text{CO})_5$, $\text{Fe}(\text{CO})_5$, $[\text{Co}(\text{CO})_3]^{3-}$, $[\text{Cr}(\text{CO})_4]^{4-}$, and $\text{Ir}(\text{CO})_3$, the total number of species isoelectronic with $\text{Ni}(\text{CO})_4$ is _____.

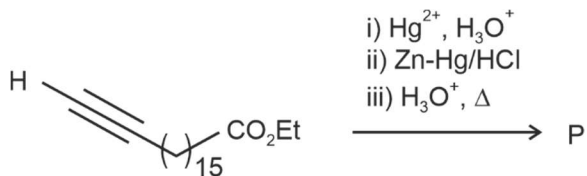
[Given atomic number : V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

Answer (1)

Sol. Total number of electron in $\text{Ni}(\text{CO})_4 = 84$

Species		Total electron
$\text{V}(\text{CO})_6$	–	107
$\text{Cr}(\text{CO})_5$	–	94
$\text{Cu}(\text{CO})_3$	–	71
$\text{Mn}(\text{CO})_5$	–	95
$\text{Fe}(\text{CO})_5$	–	96
$[\text{Co}(\text{CO})_3]^{3-}$	–	72
$[\text{Cr}(\text{CO})_4]^{4-}$	–	84
$\text{Ir}(\text{CO})_3$	–	119

12. In the following reaction sequence, the major product **P** is formed.

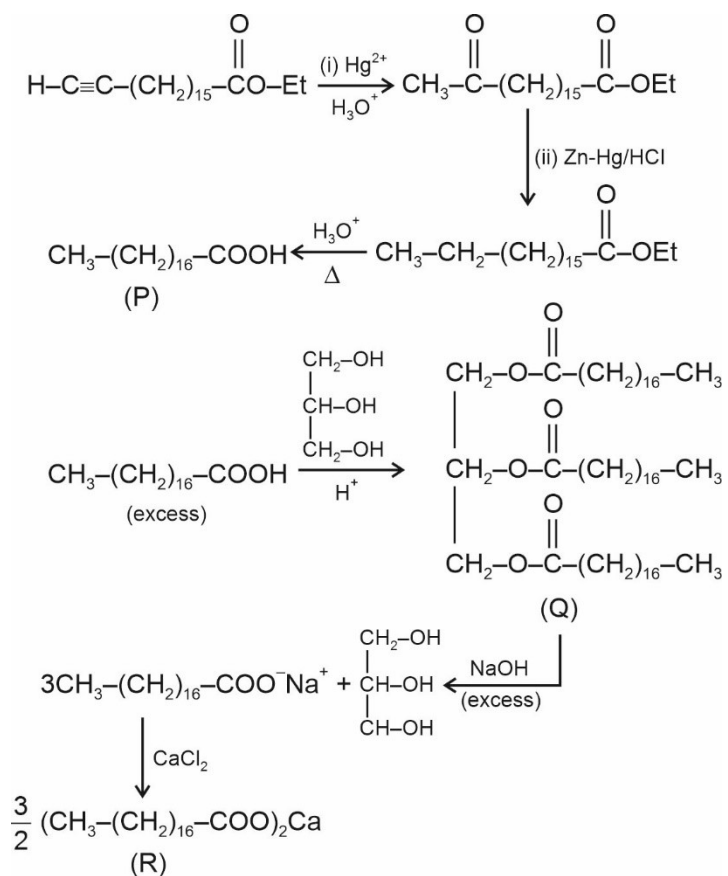


Glycerol reacts completely with excess **P** in the presence of an acid catalyst to form **Q**. Reaction of **Q** with excess NaOH followed by the treatment with CaCl₂ yields Ca-soap **R**, quantitatively. Starting with one mole of **Q**, the amount of **R** produced in gram is _____.

[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

Answer (909)

Sol.



1 mole of Q will give 1.5 mole of R.

So, mass of R produced = 606 g × 1.5
= 909 g

13. Among the following complexes, the total number of diamagnetic species is _____.



[Given, atomic number: Mn = 25, Fe = 26, Co = 27; en = $\text{H}_2\text{NCH}_2\text{CH}_2\text{NH}_2$]

Answer (1)

Sol. $[\text{Mn}(\text{NH}_3)_6]^{3+}$: Paramagnetic

$[\text{MnCl}_6]^{3-}$: Paramagnetic

$[\text{FeF}_6]^{3-}$: Paramagnetic

$[\text{CoF}_6]^{3-}$: Paramagnetic

$[\text{Fe}(\text{NH}_3)_6]^{3+}$: Paramagnetic

$[\text{Co}(\text{en})_3]^{3+}$: Diamagnetic

Only 1 complex is diamagnetic.

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

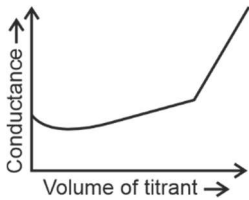
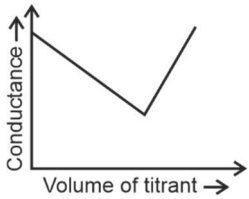
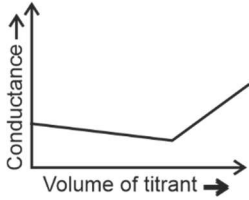
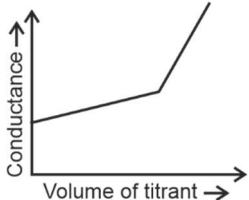
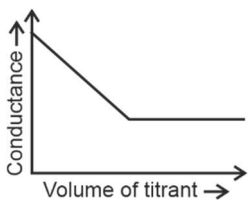
14. In a conductometric titration, small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.

The limiting ionic conductivity (Λ_0) values (in $\text{mS m}^2 \text{mol}^{-1}$) for different ions in aqueous solutions are given below:

Ions	Ag^+	K^+	Na^+	H^+	NO_3^-	Cl^-	SO_4^{2-}	OH^-	CH_3COO^-
Λ_0	6.2	7.4	5.0	35.0	7.2	7.6	16.0	19.9	4.1

For different combinations of titrates and titrants given in **List-I**, the graphs of 'conductance' versus 'volume of titrant' are given in **List-II**.

Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		List-II
(P)	Titrate: KCl Titrant: AgNO ₃	(1)	
(Q)	Titrate: AgNO ₃ Titrant: KCl	(2)	
(R)	Titrate: NaOH Titrant: HCl	(3)	
(S)	Titrate: NaOH Titrant: CH ₃ COOH	(4)	
		(5)	

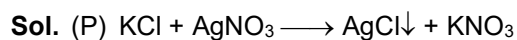
(A) P-4, Q-3, R-2, S-5

(B) P-2, Q-4, R-3, S-1

(C) P-3, Q-4, R-2, S-5

(D) P-4, Q-3, R-2, S-1

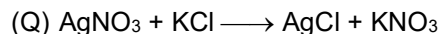
Answer (C)



Cl^- is replaced by NO_3^-

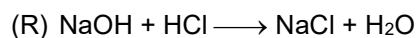
Conductance will first decrease and then after equivalence point, it will increase

$P \longrightarrow 3$

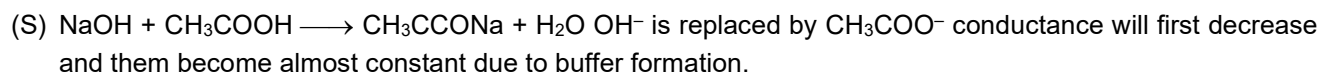


Ag^+ is replaced by K^+

Conductance will first increase slightly and then will increase further



OH^- is replaced by Cl^-



15. Based on VSEPR model, match the xenon compounds given in **List-I** with the corresponding geometries and the number of lone pairs on xenon given in **List-II** and choose the correct option.

	List-I		List-II
(P)	XeF_2	(1)	Trigonal bipyramidal and two lone pair of electrons
(Q)	XeF_4	(2)	Tetrahedral and one lone pair of electrons
(R)	XeO_3	(3)	Octahedral and two lone pair of electrons
(S)	XeO_3F_2	(4)	Trigonal bipyramidal and no lone pair of electrons
		(5)	Trigonal bipyramidal and three lone pair of electrons

(A) P-5, Q-2, R-3, S-1

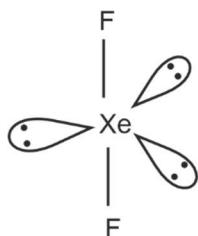
(B) P-5, Q-3, R-2, S-4

(C) P-4, Q-3, R-2, S-1

(D) P-4, Q-2, R-5, S-3

Answer (B)

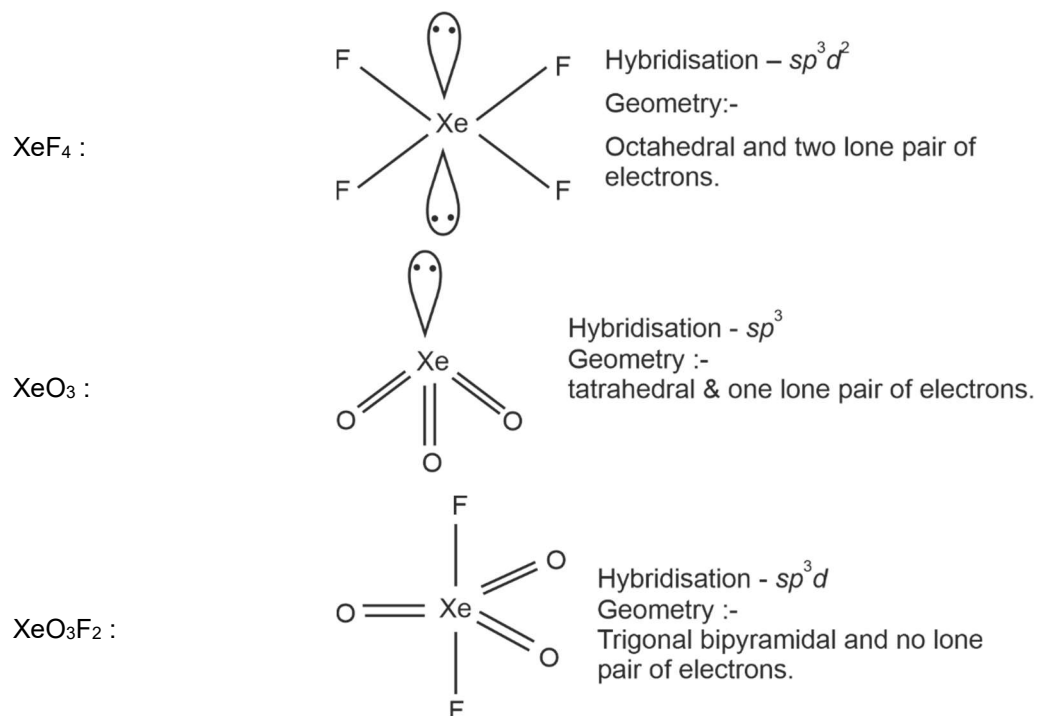
Sol. XeF_2 :



Hybridisation – sp^3d

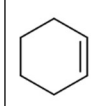
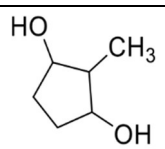
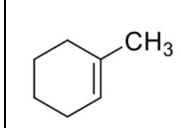
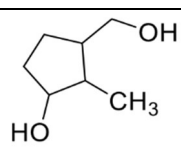
Geometry:-

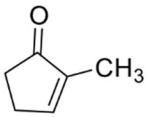
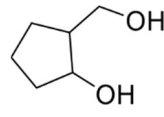
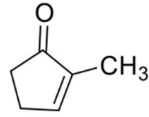
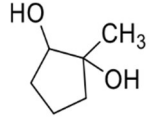
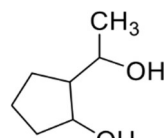
Trigonal bipyramidal and three lone pair of electrons



Correct match : P → 5; Q → 3; R → 2; S → 4

16. **List-I** contains various reaction sequences and **List-II** contains the possible products. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		List-II
(P)	 <p>i) O₃, Zn ii) aq. NaOH, Δ iii) ethylene glycol, PTSA iv) a) BH₃, b) H₂O₂, NaOH v) H₃O⁺ vi) NaBH₄</p>	(1)	
(Q)	 <p>i) O₃, Zn ii) aq. NaOH, Δ iii) ethylene glycol, PTSA iv) a) BH₃, b) H₂O₂, NaOH v) H₃O⁺ vi) NaBH₄</p>	(2)	

(R)	 i) ethylene glycol, PTSA ii) a) Hg(OAc) ₂ , H ₂ O, b) NaBH ₄ iii) H ₃ O ⁺ iv) NaBH ₄	(3)	
(S)	 i) ethylene glycol, PTSA ii) a) BH ₃ , b) H ₂ O ₂ , NaOH iii) H ₃ O ⁺ iv) NaBH ₄	(4)	
		(5)	

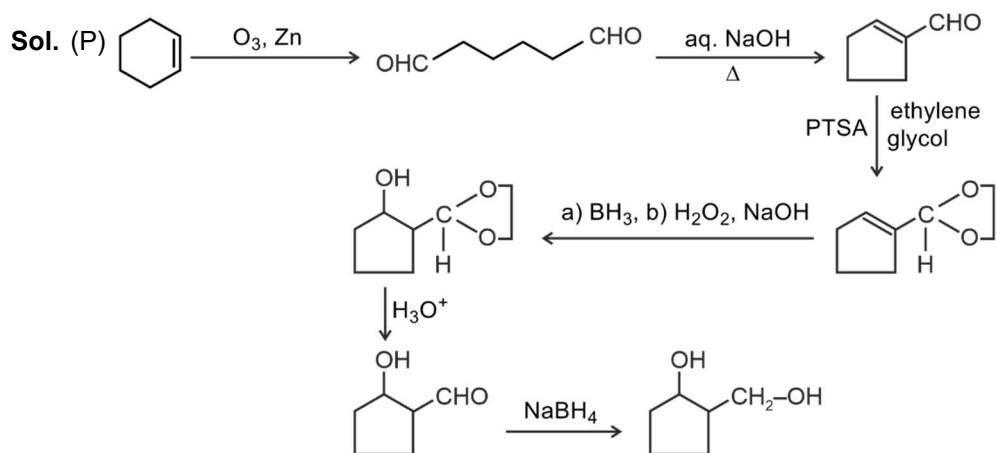
(A) P-3, Q-5, R-4, S-1

(B) P-3, Q-2, R-4, S-1

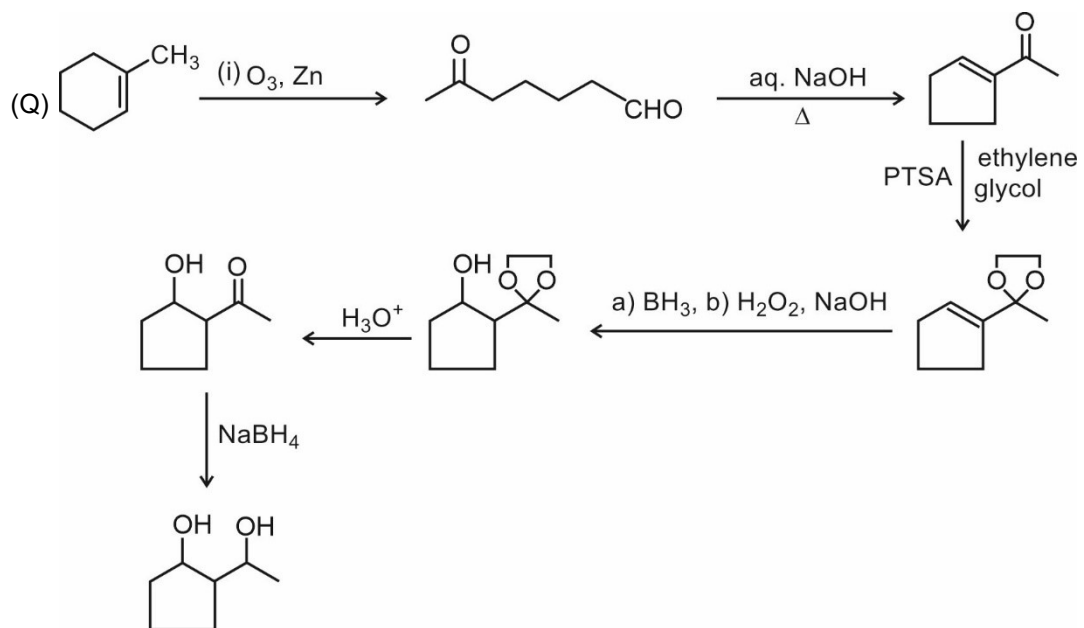
(C) P-3, Q-5, R-1, S-4

(D) P-5, Q-2, R-4, S-1

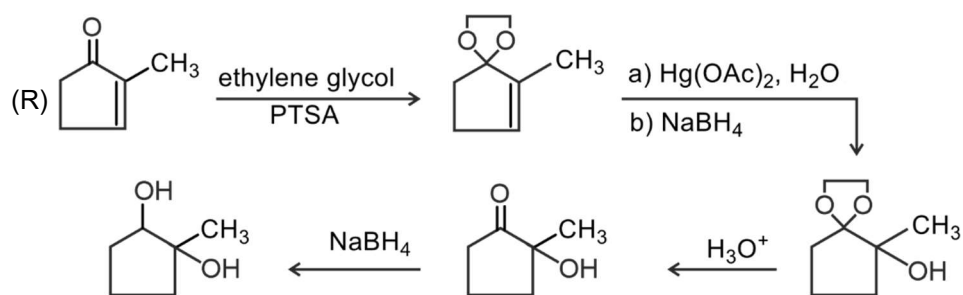
Answer (A)



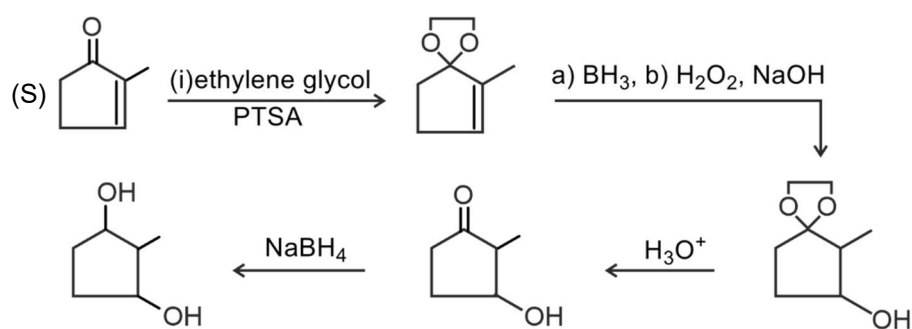
P-3



Q-5

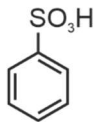
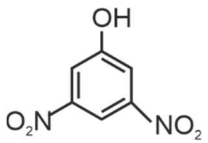
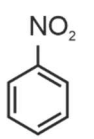
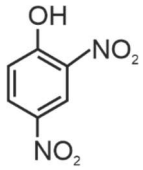
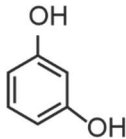
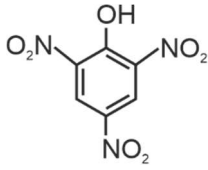
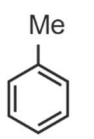
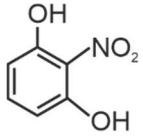
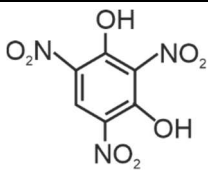


R-4



S-1

17. List-I contains various reaction sequences and List-II contains different phenolic compounds. Match each entry in List-I with the appropriate entry in List-II and choose the correct option.

	List-I		List-II
(P)	 (i) molten NaOH, H ₃ O ⁺ (ii) Conc. HNO ₃	(1)	
(Q)	 (i) Conc. HNO ₃ / Conc. H ₂ SO ₄ (ii) Sn/HCl (iii) NaNO ₂ /HCl, 0-5°C, (iv) H ₂ O (v) Conc. HNO ₃ / Conc. H ₂ SO ₄	(2)	
(R)	 (i) Conc. H ₂ SO ₄ (ii) Conc. HNO ₃ (iii) H ₃ O ⁺ , Δ	(3)	
(S)	 (i) (a) KMnO ₄ /KOH, Δ; (b) H ₃ O ⁺ (ii) Conc. HNO ₃ / Conc. H ₂ SO ₄ , Δ (iii) (a) SOCl ₂ , (b) NH ₃ (iv) Br ₂ , NaOH (v) NaNO ₂ /HCl, 0-5°C (vi) H ₂ O	(4)	
		(5)	

(A) P-2, Q-3, R-4, S-5

(B) P-2, Q-3, R-5, S-1

(C) P-3, Q-5, R-4, S-1

(D) P-3, Q-2, R-5, S-4

Answer (C)

