PART-A (PHYSICS)

01. Solution.

Let OP = r. Angular speed about the origin =

$$\omega = \frac{|V_{P_0}|_\perp}{|\vec{r}|}$$, where $|V_{P_0}|_\perp$ = The component of velocity of P w.r.t O perpendicular to OP.

$$\Rightarrow \omega = \frac{v\sin\theta}{r} \quad \text{where } r = b \operatorname{cosec}\theta$$

$$\omega = \frac{v\sin^2\theta}{b}$$

02. Solution.

Time taken in moving distance S along smooth inclined surface with inclination θ is $t_s = \sqrt{\frac{2S}{g\sin\theta}}$

And time taken in moving distance S along smooth inclined surface with inclination θ is

$$t_r = \sqrt{\frac{2S}{g(\sin\theta - \mu\cos\theta)}} = nt_s$$

Using the expression obtained above

$$\frac{1}{\eta} = \sqrt{1-\mu\cot\theta} \Rightarrow \mu\cot\theta = 1 - \frac{1}{n^2} \Rightarrow \quad \mu = \left(1 - \frac{1}{n^2}\right)\tan\theta$$

03. Solution.

The velocity of both the bodies m & M are equal. If the block M sticks to the wall, the block m will continue to move which compresses the spring through x. The K.E. of the block m will be converted into the potential energy of the spring as it compresses the spring. Conservation of energy yields

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{k}}$$

where $\frac{1}{2}(M+m)v^2 = E$

$$\Rightarrow v = \sqrt{\frac{2E}{(M+m)}} \quad \therefore x = \sqrt{\frac{2mE}{(M+m)k}}$$

04. Solution.

by conservation of linear momentum along the line of collision (along the line joining the centers of two spheres), $m_1 u \sin\theta = m_2 v$

Since, Coefficient of restitution for oblique collision, $e = -\left.\frac{v_2 - v_1}{u_2 - u_1}\right|_{\text{along the line of collision}}$

$$\Rightarrow e = \frac{v}{u\sin\theta} = \frac{m_1}{m_2} = \frac{2}{3}$$

05. Solution.

$I_{remaining} = I_{whole} - I_{removed}$

$$\text{or } I = \frac{1}{2}(9M)(R^2) - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + \frac{1}{2}m\left(\frac{2R}{3}\right)^2\right]$$

......(i)

$$\text{Here, } m = \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^3 = M$$

Substituting in Eq. (i), we have

$I = 4MR^2$

06. Solution.

The gravitational potential at a point Q $Q(OQ \equiv x)$ is given by :

$$V(x) = \begin{cases} -g_s R\left(\frac{3}{2} - \frac{1}{2}\frac{x^2}{R^2}\right), & \text{where } x < R \\ -g_s R\left(\frac{R}{x}\right), & \text{when } x > R \end{cases}$$

The energy required to project the body, to a maximum altitude of 3R from its surface, is :

$$m\left(V_B\Big|_{x=\frac{R}{2}} - V_P\Big|_{x=4R}\right) = \frac{9}{8}mg_s R$$

07. Solution.

$$h = \frac{2T\cos\theta}{r\rho g} \Rightarrow T = \frac{hr\rho g}{2\cos\theta} \quad \text{or } T \alpha \frac{h\rho}{\cos\theta}$$

$$\frac{T_w}{T_{Hg}} = \frac{h_1}{h_2} \times \frac{\cos\theta_2}{\cos\theta_1} \times \frac{\rho_1}{\rho_2}$$

Putting the values, we obtain 1 : 65

08. Solution.

$$\Delta Q = nC_p\Delta T = 2\left(\frac{3}{2}R + R\right)\Delta T$$

$$= 2\left[\frac{3}{2}R + R\right] \times = 2 \times \frac{5}{2} \times 8.31 \times 5$$

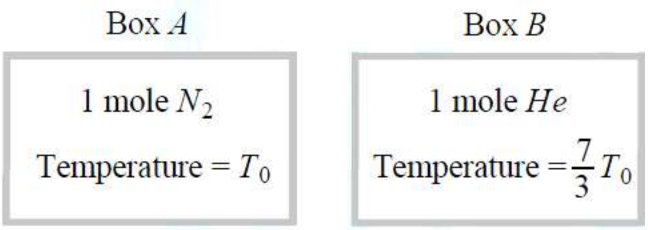
= 208 J.

09. Solution.

When two gases are mixed together then

Heat lost by the Helium gas = Heat gained by the Nitrogen gas

$$\mu_B \times (C_v)_{He} \times \left(\frac{7}{3}T_0 - T_f\right) = \mu_A \times (C_v)N_2 \times (T_f - T_0)$$



$$\Rightarrow 1\times\frac{3}{2}R\times\left(\frac{7}{3}T_0 - T_f\right) = 1\times\frac{5}{2}R\times(T_f - T_0)$$

By solving we get $T_f = \frac{3}{2}T_0$

10. Solution.

Potential energy of the particle $U = k(1-e^{-x^2})$

Force on particle $F = \frac{-dU}{dx} = -k[-e^{-x^2}\times(-2x)]$

$$F = -2kxe^{-x^2} = -2kx\left[1 - x^2 + \frac{x^4}{2!} - ......\right]$$

For small displacement F = – 2kx

$\Rightarrow F \propto -x$ i.e. motion is simple harmonic motion

11. Solution.

$$\phi(x,y,z) = ax^2$$

$$\vec{E} = -2ax\hat{i}$$

$$\int_s \vec{E}.d\vec{s} = \left(2aL\hat{i}\right)\left(-L^2\hat{i}\right) = -2aL^3$$

$$\int_s^{x=-L} \vec{E}.d\vec{s} = \left(-2a(L)\hat{i}\right).\left(L_2\hat{i}\right) = -2aL^3$$

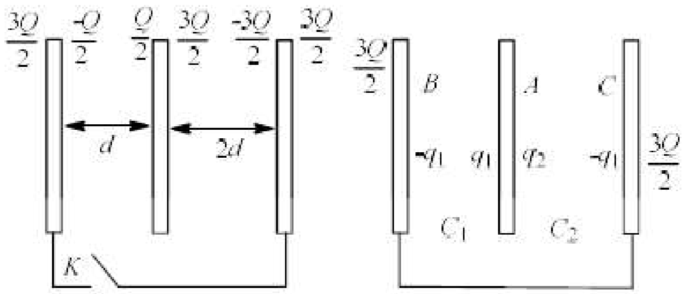
$$\int_s^{x=L} \vec{E}.d\vec{s} = 0$$

each of the other faces

$$\therefore \oint \vec{E}.d\vec{s} = -4aL^3 = \frac{q}{\epsilon_0}$$

$$\therefore q = -4a\,\epsilon_0\,L^3$$

12. Solution.



$$q_1 + q_2 = 2Q$$

$$V_{AB} = V_{AC} \Rightarrow \frac{q_1}{c_1} = \frac{q_2}{c_2}$$

$$\frac{q_1}{q_2} = \frac{c_1}{c_2} = 2 \Rightarrow q_1 = 2q_2$$

Solving, we get $q_1 = \frac{4Q}{3}, q_2 = \frac{2Q}{3}$

Charge flown through K:

$$= -\frac{Q}{2} - (-q_1) = -\frac{Q}{2} + \frac{4Q}{3} = 5Q/6$$

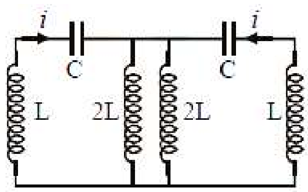
13. Solution.

Here, both Assertion and Reason are correct, and reason is the correct explanation of assertion.

14. Solution.

$$X = C\times\frac{1}{T}, C = \frac{0.4}{7\times10^{-8}} = 57K$$

15. Solution.



16. Solution.

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$R = 10\Omega, X_L = \omega L = 2000\times5\times10^{-3} = 10\Omega$$

$$X_C = \frac{1}{\omega_C} = \frac{1}{2000\times50\times10^{-6}} = 10\Omega \text{ i.e. } Z = 10\Omega$$

Maximum current $i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2A$

Hence $i_{rms} = \frac{2}{\sqrt{2}} = 1.4A$

and $V_{rms} = 4\times1.41 = 5.64V$

17. Solution.

The equation of electric field occurring in Y - direction

$$E_y = 66\cos 2\pi\times10^{11}\left(t-\frac{x}{c}\right)$$

Therefore, for the magnetic field in Z - direction

$$B_z = \frac{E_y}{c}$$

$$=\left(\frac{66}{3\times10^8}\right)\cos 2\pi\times10^{11}\left(t-\frac{x}{c}\right)$$

$$=22\times10^{-8}\cos 2\pi\times10^{11}\left(t-\frac{x}{c}\right)$$

18. Solution.

$$\because \beta - \frac{D\lambda}{d} \Rightarrow \text{for } \beta_{max}, \text{'d' will be min} - 2d_0 - d_0 - d$$

And for β_{min},'d' will be max = $2d_0$ | $d_0 = 3d_0$

$$\because \beta_{max} = \frac{D\lambda}{d} \Rightarrow \text{and } \beta_{min} = \frac{D\lambda}{3d_0}$$

$$\because \beta_{max} - \beta_{min} = \frac{D\lambda}{d_0}\left(1-\frac{1}{3}\right)\ \left(\frac{2D\lambda}{3d_0}\right)$$

19. Solution.

Conceptual

20. Solution.

Young's modulus $Y = \frac{FL}{Al} = \frac{4FL}{\pi d^2 l}$

$$=\frac{(4)(1.0\times10.0)(2)}{\pi(0.4\times10^{-3})^2(0.8\times10^{-3})}$$

= 2.0×10^{11} NM^{-2}

Further, $\frac{\Delta Y}{Y} = 2\left(\frac{\Delta d}{d}\right)+\left(\frac{\Delta l}{l}\right)$

$$\therefore \quad \Delta Y = \left\{2\left(\frac{\Delta d}{d}\right)+\left(\frac{\Delta l}{l}\right)\right\}Y$$

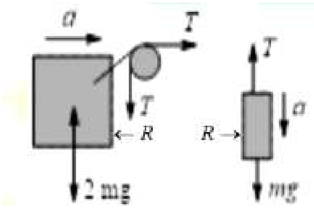
$$=\left\{2\times\frac{0.01}{0.4}+\frac{0.05}{0.8}\right\}\times2.0\times10^{11}$$

= $0.2 + 10^{11}$ Nm^{-2}

Or $(Y\pm\Delta Y) = (2\pm0.2)\times10^{11}\,Nm^{-2}$

21. Solution.

T–R = 3ma and R = ma $\Rightarrow$ T = 4ma

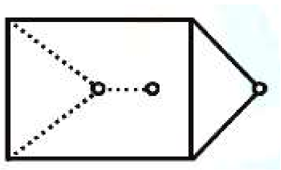


mg–T = ma

$$a = \frac{g}{5} = 2$$

Hence acceleration of B w.r.t ground is $2\sqrt{2}$ m/s^2.

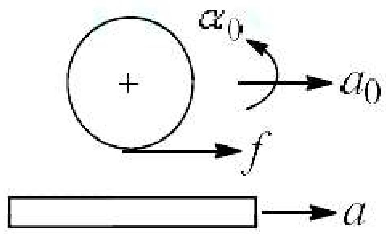
22. Solution.



$$\bar{x} = \frac{(M\times0)+\left(-\frac{M}{4}\times\frac{-L}{3}\right)+\left(\frac{M}{4}\times\frac{4L}{6}\right)}{M} = \frac{L}{4}$$

23. Solution.

Let the frictional force be in the forward direction, then



$f = Ma_0$

and $fR = \frac{MR^2}{2}\alpha_0$

$a_0 = \frac{f}{M}$ and $\alpha_0 = \frac{2f}{MR}$

For pure rolling, $a_0 + R\alpha_0 = a$

$\therefore \frac{f}{M}+\frac{2f}{M} = a \Rightarrow f = \frac{Ma}{3}$

For pure rolling, $f \le fL = \mu Mg$

$\frac{Ma}{3} \le \mu Mg \Rightarrow a \le 3\mu g \therefore a_{max} = 9m/s^2$

24. Solution.

Slope of line $A = \frac{(1006-1000)mm}{T°C} = \frac{\Delta L}{\Delta T} = L\alpha_A$

i.e., $\frac{6}{T}$ mm/°C = (1000 mm) αA (i)

Similarly, for line B,

$\frac{2}{T}$ mm/°C = (1002 mm) αB (ii)

Dividing Eq. (i) by Eq.(ii),

$3 = \frac{1000\alpha_A}{1002\alpha_B} = \alpha_A = 3\alpha_B$ (iii)

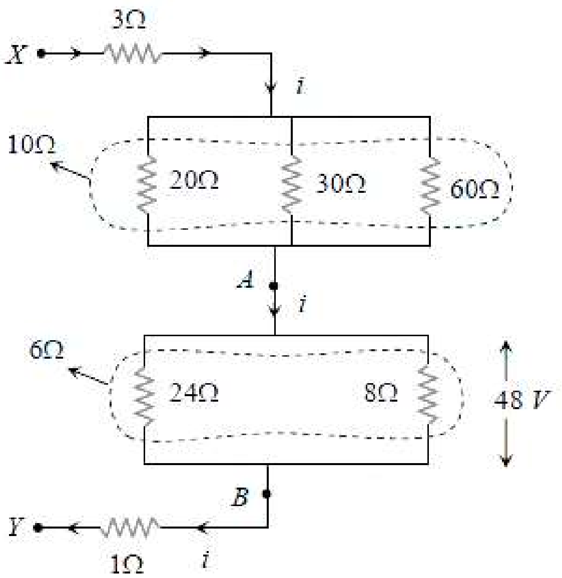
From Eq. (iii), αA = 3 × 9 × 10^{-6} = 27 × 10^{-6}/°C

25. Solution.

As the tension in string Y is increased hence it's frequency will increase. But as given, beat frequency is decreased so , in the beginning $n_X - n_Y$ = 4 $\Rightarrow$ 300 – n_Y = 4 $\Rightarrow n_Y$ = 296 Hz

26. Solution.

The given circuit can be redrawn as follows



Resistance between A and B = $\frac{24\times 8}{32} = 6\Omega$

Current between A and B = Current between X and Y = $i = \frac{48}{6} = 8A$

Resistance between X and Y = (3 + 10 + 6 + 1) = 20 Ω

$\Rightarrow$ Potential difference between X and Y = 8 × 20 = 160V

27. Solution.

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{f_1} = \frac{(\mu_1 - \mu)}{\mu}\left[\frac{2}{R}\right] = \frac{(\mu_1 - 4)}{4}\left[\frac{2}{R}\right] = \frac{3\mu_1 - 4}{2R}$$

$$\frac{1}{f_2} = -\left[\frac{\mu_2 - \mu}{\mu}\left(\frac{2}{R}\right)\right] = -3\left[\frac{\mu_2 - \mu}{2R}\right]$$

$$\frac{1}{24} = \frac{3(\mu_1 - \mu_2)}{2R} \Rightarrow \mu_1 - \mu_2 = \frac{2R}{24\times 3} = \frac{R}{12\times 3} = \frac{20}{36} = \frac{5}{9}$$

28. Solution.

As the current depends on the number of photons incident. Now by inverse square law,

$$12 \propto \frac{1}{(0.2)^2} \text{ and } 1 \propto \frac{1}{(0.4)^2}$$

$$\therefore \frac{l}{12} = \frac{(0.2)^2}{(0.4)^2} = \frac{1}{4}$$

$$\text{or } I = \frac{12}{4} = 3\text{mA}$$

29. Solution.

Current in R_1=5/500=10mA and current in R_2=10/1500=20/3 mA , hence the current in Zener Diode=10/3 mA

30. Solution.

Since there is no shunt resistance, I = 9 × 60 μA

$$\therefore \frac{6}{11000 + G} = 540\times 10^{-6} \Rightarrow G = \frac{1000}{9}$$

Since for half deflection,

$$G = \frac{RS}{R-S} \Rightarrow \frac{1000}{9} = \frac{11000S}{11000 - S} \Rightarrow S = 110\Omega$$

PART-B CHEMISTRY

31. A) SO_3 & CO_3 ; Both are sp^2 & planar triangular

B) SO_3^{2-} & NH_3 ; Both are sp^3 & pyramidal

C) $PCl_5 : sp^3d$ & trigonal bipyramidal

$POCl_3 : sp^3$ & tetra hedral

D) $XeF_2 : sp^3d$ & linear

$ClF_3 : sp^3d$ & T–shape

32. Sol. B_2 : By distribution 10 electrons only two electrons in π - B.M.O are extra left without cancelling with A.B.M.O electrons

N_2 : By distribution 14 electrons only 4 electrons & 2 electrons in B M O & extra left without cancelling with ABM O electrons

35. Sol.

0.1 moles of the complex – 28.7 g of AgCl
1 mole gives of complex – 287 g of AgCl
– 2 moles of AgCl

$\Rightarrow$ 2 Cl^- ions should be ionisable.

36. Sol. The complex cannot show hydration isomerism as no H_2O ligands are present.

37. Sol. The colour of $KMnO_4$ is due to charge transfer phenomenon

38. Sol.

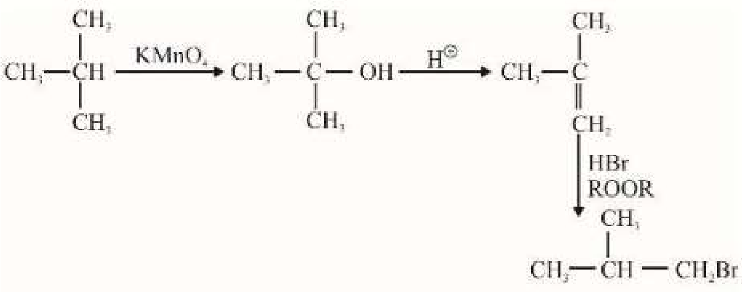
$$n_{m.eq} NH_3 = n_{m.eq} H_2SO_4$$

$$= 10 \times 1 \times 2 = 20 \, m\,eq\,of\;NH_3 = 20 \text{ m mol of } NH_3$$

$$\%N = \frac{1400 \times n_{eq} NH_3}{wt.of\ organic\,compound}$$

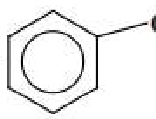
$$= \frac{1400 \times 20 \times 10^{-3}}{0.5} = 56\% .$$

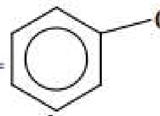
39. Sol.

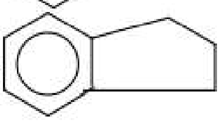


40. Sol. Reactivity order IV > I > III > II > V on the basis of R and I effect of associated groups.

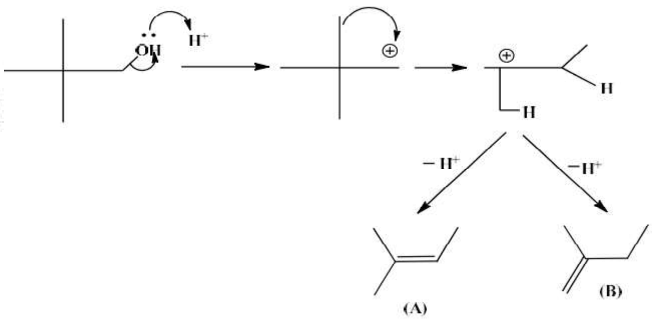
41. Sol.

A=  $-CH_2-CH=CH_2$ Friedal- craft's aklylation

B=  $-CH_2CH_2CH_2OH$ Hydroboration -oxidation

C= 

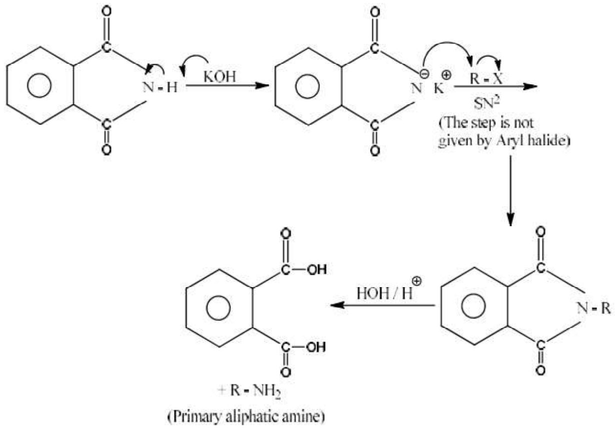
42. Sol.



43. Sol. Cleavage of the double bond by Ozonolysis, iodoform Rxn, dry distillation of calcium salts to give cyclopentanone, followed by wolf–kishner reduction to give cyclohexane.

44. Sol. Benzyllic oxidation to give potassium salt of Benzoic acid, followed by acidification to give Benzoic acid.

45. Sol. Gabriel pthalamide synthesis



46. Sol. Keratin and myosin are fibrous proteins and insoluble in H_2O.

47. Sol.

$$(61.9 + 76.3) = \frac{1.382 \times 10^{-4} \times 1000}{S}$$

$$\therefore S = 10^{-3} M .$$

49. Sol.

Required energy $= I_1 + I_2$

$$I_1 = 24.6eV$$

$$I_2 = I_H \times Z^2 = 13.6 \times 2^2 = 54.4eV$$

$$\therefore E = 24.6 + 54.4 = 79eV$$

50. Sol.

$$3A \quad \rightarrow B$$

$$t = 4\,\text{min}; a - 3x \quad = \quad x$$

$$\Rightarrow 4x = a \Rightarrow x = \frac{a}{4}$$

$\therefore$ At 4 min 75% of first order is completed.

$$\therefore t_{75\%} = \frac{2t_1}{2} \Rightarrow \frac{t_1}{2} = 2\,\text{min}.$$

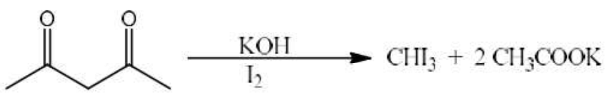
51. Sol.

$$X = 12(Mg);\ Y = 15(P)$$

54. Sol.

Except $-\overset{\overset{\large O}{\|}}{C}-O-R$, remaining are ring activating groups.

55. Sol.



57. Sol.

$$H_{2(g)} \rightarrow 2H^+_{(aq)} + 2e^-$$

$$0.413 = -\frac{0.059}{2}\log\left[H^+\right]^2$$

$$\Rightarrow pH = 7.$$

58. Sol.

$$E^0 = E^0_{Ag^+/Ag} + 0.06\log K_{SP}$$

59. Sol.

Number of moles of $CH_3COOH = 0.25 \times 0.3 = 0.075\,moles$

Number of moles of $CH_3COO^- = 0.56 \times 0.3 = 0.168$

$\therefore$ Number of moles of CH_3COO^- left $= 0.168 - 0.006 = 0.162$

Final number of moles of $CH_3COOH = 0.075 + 0.006 = 0.081$

$\therefore pH = 4.7 + \log\frac{0.162}{0.081}$.

PART-C (MATHEMATICS)

61. Solution.

Given set A with equation $|x+1|<2$ and set B with equation $|x-1| \ge 2$.

$A : x \in (-3, 1)$

$B : x \in (-\infty, -1] \cup [3, \infty)$

Take, $B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$ also,

$A \cap B = (-3, -1)$

62. Solution.

We have $R = \{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$

$R^{-1} = \{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$

Hence, $RoR^{-1} = \{(3,3),(3,5),(5,3),(5,5)\}$

63. Solution.

$C.I.$	f_i	x_i	$f_i x_i$	$C.F.$
0-6	a	3	$3a$	a
6-12	b	9	$9b$	$a+b$
12-18	12	15	180	$a+b+12$
18-24	9	21	189	$a+b+21$
24-30	5	27	135	$a+b+26$
	$N = (26+a+b)$		$(504+3a+9b)$	

$$\text{Mean } \frac{504 + 3a + 9b}{26 + a + b} = \frac{309}{22}$$

$$\Rightarrow \quad 243a + 111b = 3054$$

$$\Rightarrow \quad 81a + 37b = 1018 \text{...........(i)}$$

Medain class is $12 - 18$

$$\text{Now, median } 12 + \frac{\frac{a+b+26}{2} - (a+b)}{12} \times 6 = 14$$

$$\Rightarrow \frac{a+b+26+2a-2b}{2} = 4 \Rightarrow a + b = 18 \quad \text{............(ii)}$$

On solving eqs. (i) and (ii), we get $a = 8$, $b = 10$

64. Solution.

$$\mu = \sum x_i P(x = x_i) = 3.24$$

65. Solution.

$$f(x) = -\int_0^x f(t)\tan t dt + \int_0^x \tan(t-x)dt$$

$$f(x) = -\int_0^x f(t)\tan t dt + \int_0^x \tan(-t)dt$$

$(\text{use } f(a-x) = f(x))$

$$f(x) = -\int_0^x f(t)\tan t dt = \int_0^x \tan(t)dt$$

Differentiating w.r.t x, we have

$f(x) = -f(x)\tan x - \tan x$

$$\frac{dy}{dx} = -(\tan x)y - \tan x$$

$$\frac{dy}{dx} + (\tan x)y = -\tan x$$

$$IF = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

Therefore, solution is y. sec x =

$$\sec x = -\int \sec x \tan x c$$

or y sec x = – sec x + c

or **y = c cos x – 1**

Curve passes through (0,0)

∴ 0 = c – 1 or c = 1

∴ y = cos x – 1

Therefore, the maximum value of y is 0.

66. Solution.

We have

In S_1 units place can have 1 or 3 or 5

In S_2 it is $5C_3 \times 3 \times \frac{4!}{2!} + 5C_2(9 \times 2 + 6) = 600$

67. Solution.

Let the probability of the faces 1,3,5 or 6 is p for each face.

Hence the probability of the faces 2 or 4 is 3p, therefore

$$4p + 6p = 1 \Rightarrow p = \frac{1}{10}$$

$$P(1) = P(3) = P(5) = P(6) = \frac{1}{10}$$

$$P(2) = P(4) = \frac{3}{10}$$

P (total of 7 with a draw of dice) = P (16,61,25,52)

$$= 2\left(\frac{1}{10}.\frac{1}{10}\right) + 2\left(\frac{3}{10}.\frac{1}{10}\right) + 2\left(\frac{3}{10}.\frac{1}{10}\right)$$

$$= \frac{2+6+6}{100} = \frac{14}{100} = \frac{7}{50}$$

68. Solution.

Apply condition for externally touching circle.

69. Solution.

The hyperbola $\left(\frac{x-\sqrt{2}}{4}\right)^2 - \frac{\left(y+\sqrt{2}\right)^2}{2} = 1$

$$a = 2, b = \sqrt{2}, e = \sqrt{\frac{3}{2}}$$

If D be the centre, then

$DC = ae = \sqrt{6}$ and $DA = a = 2$

$AC = \sqrt{6} - 2$ and $BC = \frac{b^2}{a} = 1$

Now area of $\Delta ABC = \frac{1}{2}(AC)(BC) = \sqrt{\frac{3}{2}} - 1$

70. Solution.

Use $\bar{r} \cdot \bar{a} = 0$

$\bar{r} \cdot \bar{b} = 0$

where $\bar{v} = x\hat{i} + y\hat{j} + z\hat{k}$

And $\bar{x} \cdot \hat{i} = 21$

71. Solution.

For option (1) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = P(A) = \frac{1}{3}$

Similarly $P\left(\frac{A'}{B'}\right) = P(A') = \frac{2}{3}$

$$P\left(\frac{A}{B'}\right) = \frac{P(A)(1-P(B))}{(1-P(B))} = \frac{\frac{1}{3}.\frac{5}{6}}{\frac{5}{6}} = \frac{1}{3}$$

$$P\left(\frac{A}{A \cup B}\right) = \frac{P\left(A \cap (A \cup B)\right)}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A \cup B)} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6} - \frac{1}{18}}$$

$$= \frac{6}{6+3-1} = \frac{3}{4}$$

72. Solution.

$$x^2 + 5^{\frac{1}{2}} = -(20)^{\frac{1}{4}} x \Rightarrow \left(x^2 + \sqrt{5}\right)^2 = \sqrt{20}\, x^2$$

$$\Rightarrow x^4 + 5 + 2\sqrt{5}x^2 = 2\sqrt{5}x^2 \Rightarrow x^4 - 5 \Rightarrow x^8 = 25$$

So, $\alpha^8 + \beta^8 = 50$

73. Solution.

If we write the elements of A + A. We can certainly find 39 distinct element as

$1 + 1, 1 + a_1, 1 + a_2 \ldots\ldots 1 + a_{18}, 1 + 77, a_1 + 77, a_2 + 77, \ldots a_{18} + 77, 77 + 77$

It means all other sums are already present in these 39 values, which is only possible in case when all numbers are in A.P.

Let the common difference be 'd'

$77 = 1 + 19\,d \Rightarrow d = 4$

So, $\sum_{i=1}^{18} a_1 = \frac{18}{2}[2a_1 + 17d] = 9[10 + 68] = 702$

PQ is focal chord.

74. Solution.

$\overline{\beta_1} = t\overline{\alpha} \quad \overline{\alpha}.\overline{\beta_2} = 0$

$\overline{\beta} = \overline{\beta_1} + \overline{\beta_2}$

Taking dot with $\overline{\alpha}$

$\overline{\alpha}.\overline{\beta} = \overline{\alpha}.\overline{\beta_1} + \overline{\alpha}.\overline{\beta_2}$

$4 + 6 - 20 = \overline{\alpha}.t\overline{\alpha} + 0 \quad -10 = t\left|\overline{\alpha}\right|^2$

$-10 = t(16 + 9 + 25) \quad \therefore t = \frac{-1}{5}$

$\therefore \overline{\beta} = -\frac{1}{5}\overline{\alpha} + \beta_2 \qquad \therefore \overline{\beta_2} = \overline{\beta} + \frac{1}{5}\overline{\alpha}$

$= \left(\bar{i} + 2\bar{j} - 4\bar{k}\right) + \frac{1}{5}\left(4\bar{i} + 3\bar{j} + 5\bar{k}\right) \qquad = \frac{9\bar{i} + 13\bar{j} - 15\bar{k}}{5}$

$5\overline{\beta_2}.\left(\bar{i} + \bar{j} + \bar{k}\right) = \left(9\bar{i} + 13\bar{j} - 15\bar{k}\right).\left(\bar{i} + \bar{j} + \bar{k}\right) \quad = 9 + 13 - 15 = 7$

75. Solution.

$\{x+b\} = \{x\}$ where b is an integer

$$\therefore [x] + \sum_{b=1}^{1000} \frac{\{a+b\}}{1000} = [x] + \frac{1000\{x\}}{1000}$$

$= [x] + \{x\} = x$

76. Solution.

A (1, 0, 7) B (1, 6, 3)

Midpoint of AB= (1, 3, 5) lies in the line

DR's of AB (0, 6, -4) the line passing through A and B is perpendicular to the given line hence B in the mirror image.

Statement II is also true but not a correct explanation of I as there are infinitely many lines passing through the midpoint of the line segment and one of the lines is perpendicular bisector

77. Sol.

$$\frac{20!}{p!q!r!}(2x)^p(-y)^q(z)^r = \frac{20!}{p!q!r!}2^p(-1)^q x^p y^q z^e$$

$p + q + r = 20 \qquad q = 0$

$p + r = 20$ p भि खपक्व r ायूपभ नक्ष

भि + ायूपभ = भि (मज्पज गनजब नपक मिचप नक)

∴ ह्त्रापपबमुप्धम्स नक्ष

78. Sol. Put $Z = x + iy$

$x^2 - y^2 + 2ixy + \alpha x + \alpha iy + \beta = 0$

Put $y = \pm 2$

$x^2 - 4 + \alpha x + \beta + i\,(2x + y + \alpha y) = 0$

So

$x^2 + \alpha x + \beta - 4 = 0$

and

$2xy + \alpha y = 0$

$\alpha = -2x$ Put

$x^2 - 2x^2 + \beta - 4 = 0$

$x^2 = \beta - 4$

$\beta \ge 4$

Minimum value of $\beta = 4$

79. Solution.

Here, $3\sin^2\theta = \cos 2\phi$ and $3\sin\theta.\cos\theta = \sin 2\phi$.

Squaring and adding, $9\sin^2\theta(\sin^2\theta + \cos^2\theta) = 1$

i.e. $\sin\theta = \frac{1}{3}$ and $\cos\theta = \frac{2\sqrt{2}}{3}$

$\therefore \cos 2\phi = 3.\frac{1}{9} = \frac{1}{3}$ and $\sin 2\phi = \frac{2\sqrt{2}}{3}$

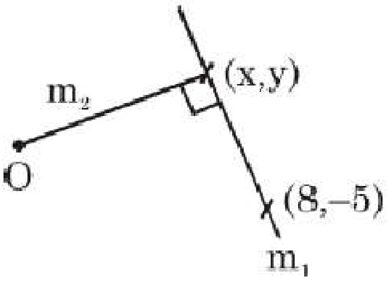
$\therefore \cos(\theta + 2\phi) = \cos\theta.\cos 2\phi - \sin 2\phi$

$= \frac{2\sqrt{2}}{3}.\frac{1}{3} - \frac{1}{3}.\frac{2\sqrt{2}}{3} = 0$ and $\theta + 2\phi < \frac{3\pi}{2}$

$\therefore \theta + 2\phi = \frac{\pi}{2}$

80. Solution.

$$\left.\begin{array}{l} x+y-3=0 \\ 2x+3y-1=0 \end{array}\right\}(8,-5)$$



$m_1m_2=-1$

$$\Rightarrow\left(\frac{y+5}{x-8}\right)\left(\frac{y}{x}\right)=-1$$

$\Rightarrow x^2+y^2-8x+5y=0$

81. Solution.

$f'(x)=3x^2+3>0 \quad \therefore$ f(x) is increasing

$$\frac{a}{1-r}=f(3)=27 \quad f'(0)=3; a(1-r)=3$$

$$a^2=81 \quad \Rightarrow a=9 \quad r=\frac{2}{3}$$

82. Solution.

$=3(1)+5(2)+7(3)+9(4)+11(5)+13(6)+$
$15(7)+17(8)+19(9)+21(10)$

$$=\sum_{r=1}^{10}(2r+1)r=825$$

83. Solution.

$$\underset{x\to0}{\mathrm{Lt}}\, f(x)=\underset{x\to0}{\mathrm{Lt}}\frac{\left(\frac{2^x-1}{x}\right)^3}{(\log 2)\left(\frac{\sin(x\log 2)}{x\log 2}\right)\frac{\log(1+x^2\log 4)}{x^2\log 4}.\log 4}$$

$$=\frac{(\log 2)^3}{(\log 2)(2\log 2)}=\frac{1}{2}\log 2$$

$$\therefore\left[\frac{1}{2}\log 2\right]=0$$

84. Solution.

$a^2e^2=36$

$\Rightarrow a^2-b^2=36 \quad .....(1); 4ab=?$

Using $r=(s-a)\tan\frac{A'}{2}$ in ΔOCF

1 = (s–a) tan45° where a = CF $\qquad$ 2=2 (s–a)

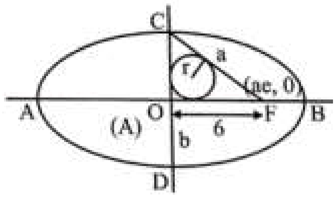
Or $\quad 2=2s-2a=2s-AB$

Or $\quad 2=(OF+FC+CO)-AB$

$$2=6+\frac{AB}{2}+\frac{CD}{2}-AB$$

$$\frac{AB-CD}{2}=4\Rightarrow 2(a-b)=8\Rightarrow a-b=4$$

....................(2)



From (1) and (2)

$a+b=9\Rightarrow 2a=13; 2b=5\Rightarrow(AB)(CD)=65$

85. Solution.

Let $f(x)=x^3+ax^2+bx+c$

$f'(x)=3x2+2ax+b \qquad \Rightarrow f'(1)=3+2a+b$

$f''(x)=6x+2a \qquad \Rightarrow f''(2)=12+2a$

$f'''(x)=6 \qquad \Rightarrow f'''(3)=6$

$\because f'(1)=x^3+f'(1)x^2+f''(2)x+f'''(3)$

$\Rightarrow f'(1)=a\Rightarrow 3+2a+b=a \qquad \Rightarrow a+b=-3$

......(1)

$\Rightarrow f''(2)=b\Rightarrow 12+2a=b \qquad \Rightarrow 2a-b=-12$

......(2)

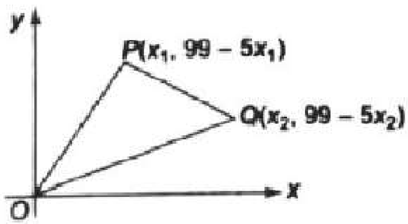
From (1) and (2)

$3a=-15\Rightarrow a=-5\Rightarrow b=2$

$\Rightarrow f(x)=x^3-5x^2+2x+6$

$\Rightarrow f(2)=8-20+4+6=-2$

86. Solution.



$$\text{Area}=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x_1 & 99-x_1 & 1 \\ x_2 & 99-x_2 & 1 \end{vmatrix}$$

$= \frac{1}{2}\left|\left[(x_1 - x_2)99\right]\right|$

Area is an integer. Then both x_1 and x_2 are simultaneously either even or odd.

Hence, ${}^{10}C_2 + {}^{10}C_2 = 2.{}^{10}C_2 = 90$

87. Solution.

K = 3! × 3! × 1 = 36

P = 6! × 3! × 6! = 6! × 7 = 5040

88. Solution.

The probability of drawing one white balls and one green ball from the first urn is $\frac{1}{5}$

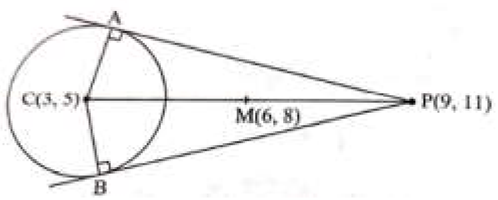
The probability of drawing one white ball and one green ball from the second urn is $\frac{1}{3}$

The probability of drawing one white ball and one green ball from the third urn is $\frac{2}{11}$,

Therefore, the probability that the third urn was chosen is $\frac{\frac{2}{11}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}} = \frac{15}{59} = \frac{a}{b}$

Hence 15 + 59 = 74

89. Solution.



The point inside the quadrilateral ACBP which is equidistant from all the four vertices is the centre M (6.8) of the circle described on PC as diameter.

Hence , distance from origin to the point M is

$\sqrt{36+34} = \sqrt{100} = 10$

90. Solution.

f(–x) = f(x), g(x), g(x) g(–x)=1

$\int_0^a f(x)dx = 10 \qquad I = \int_{-ax}^{a} \frac{f(x)}{1+g(x)} dx$

Using King $I = \int_{-a}^{a} \frac{f(x)}{1+g(x)} \quad \Rightarrow I = \int_{-a}^{a} \frac{g(x)f(x)dx}{1+g(x)}$

Eqn. (1) + Eqn. (2)

$\Rightarrow \quad 2I = \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \Rightarrow I = \int_0^a f(x)dx = 10$

(JTS - MOTION)- ANSWER KEY 13-01-2024																			
1	C	**2**	A	**3**	B	**4**	C	**5**	A	**6**	C	**7**	C	**8**	D	**9**	B	**10**	D
11	C	**12**	A	**13**	A	**14**	A	**15**	B	**16**	D	**17**	D	**18**	C	**19**	A	**20**	B
21	28	**22**	25	**23**	9	**24**	27	**25**	296	**26**	160	**27**	5	**28**	3	**29**	20	**30**	110
31	D	**32**	C	**33**	B	**34**	C	**35**	B	**36**	D	**37**	C	**38**	C	**39**	D	**40**	B
41	C	**42**	D	**43**	B	**44**	B	**45**	C	**46**	C	**47**	C	**48**	A	**49**	B	**50**	B
51	3	**52**	8	**53**	4	**54**	6	**55**	1	**56**	2	**57**	7	**58**	2	**59**	5	**60**	2
61	C	**62**	C	**63**	A	**64**	A	**65**	C	**66**	B	**67**	C	**68**	A	**69**	B	**70**	
71	C	**72**	B	**73**		**74**	D	**75**		**76**	C	**77**	A	**78**	C	**79**	A	**80**	C
81	5	**82**	825	**83**	0	**84**	65	**85**	2	**86**	90	**87**	540	**88**	74	**89**	10	**90**	10