Shri. A. R. Tambe Sir's

SHRADDHA GROUP OF INSTITUTIONS

ONLINE TEST SERIES

SAMPLE QUESTION PAPER

SHRADDHA SARVOTTAM : JEE-MAIN

IMPORTANT INSTRUCTIONS

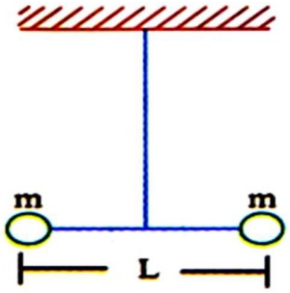
1. On the Answer Sheet, fill in the particulars on Side-1 and Side-2 carefully with blue/black ball point pen only.
2. The test is of 3 hours duration and this Test Booklet contains 75 questions. Each question carries 4 marks. For each correct response, the candidate will get 4 marks. For each incorrect response, one mark will be deducted from the total scores. The maximum marks are 300.
3. In this Test Paper,Each Subject will have two sections. Section A will be of Multiple-Choice Questions (MCQs) and Section B will contain Questions whose answers are to be filled in as a numerical value. There will be negative marking for incorrect answer in Section A and Section B. For each question in Section B, a candidate has to enter the correct integer value of the answer using the mouse and the on- screen virtual numeric keypad in the place designated to enter the answer. For Section B, the answer should be rounded off to the nearest integer.
4. In case of more than one option correct in any question, the best correct option will be considered as answer.
5. Use Blue/Black Ball Point Pen only for writing particulars on this page/marking responses.
6. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
7. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator before leaving the Room/Hall. The candidates are allowed to take away this Test Booklet with them.
8. The candidates should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet. Do not write your Form No. anywhere else except in the specified space in the Test Booklet/Answer Sheet.
9. Use of white fluid for correction is not permissible on the Answer Sheet.

SUBJECT : PHYSICS

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

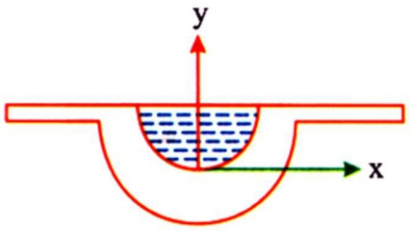
1) Two small balls each of mass m are connected by a light rigid rod of length L. The system is suspended from its centre by a thin wire of torsional constant k. The rod is rotated about the wire through an angle θ_0 and released. The tension in the rod as the system passes through the mean position is



A) $\frac{\sqrt{k^2\theta_0^4 + 4m^2g^2L^2}}{2L}$ B) $\frac{\sqrt{k^2\theta_0^4 + m^2g^2L^2}}{L}$

C) $\frac{k\theta_0^2}{2L}$ D) $\frac{k\theta_0^2 + mgL}{L}$

2) A small hole is made at the bottom of a symmetrical jar as shown in figure. A liquid is filled in to the jar up to a certain height. The rate of dissension of liquid is independent of level of the liquid in the jar. Then the surface of jar is a surface of revolution of curve as represented mathematically



A) $y = kx^4$ B) $y = kx^2$
C) $y = kx^3$ D) $y = kx^5$

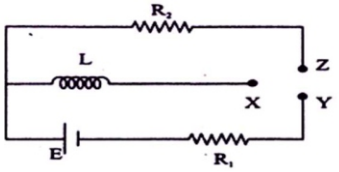
3) The electric field of an electromagnetic wave changes with the time as $E = K(1+\cos\Omega t)\cos\omega t$, where $\Omega = 5 \times 10^{15} s^{-1}$, $\omega = 2 \times 10^{16} s^{-1}$ and K is constant. This radiation is incident on a sample of hydrogen atoms initially in ground state. Assume that atoms absorb light as photons. Neglecting recoil of hydrogen nucleus on ionisation, what will be the energy of ejected electrons from hydrogen. [The ionisation energy of hydrogen atom = 13.6 eV and $h = 2\pi \times 6.6 \times 10^{-16}$ eV-s]

A) 0.7 eV B) 0.9 eV
C) 1.4 eV D) 2.9 Ev

4) Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as

A) 2.123 cm B) 2.125 cm
C) 2.121 cm D) 2.124 cm

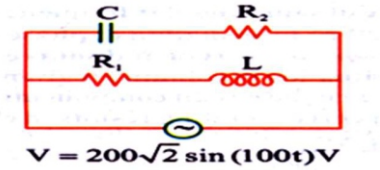
5) In the circuit shown, X is joined to Y for a long time and then X is joined to Z. The total heat produced in R_2 is



A) $\frac{LE^2}{2R_1^2}$ B) $\frac{LE^2}{2R_2^2}$

C) $\frac{LE^2}{2R_1R_2}$ D) $\frac{LE^2R_2}{2R_1^3}$

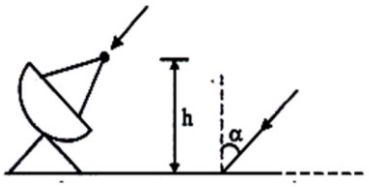
6) In the circuit shown in figure: $R_1 = 10\Omega,\ L = \frac{\sqrt{3}}{10} H, R_2 = 20\Omega$ and $C = \frac{\sqrt{3}}{2}$ mF.
Current L-R_1 circuit is in C - R_2 circuit is I_2 and the main current is I



Phase difference between I_1 and I_2

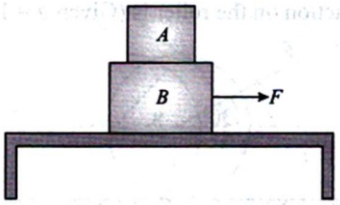
A) 0° B) 90°
C) 180° D) 60°

7) Radio waves coming at angle α to vertical are received by a ladder after reflection from a nearby water surface and also directly. What can be height of antenna from water surface so that it records a maximum intensity (a maxima) (wavelength = λ)



A) $\frac{\lambda}{2\cos\alpha}$ B) $\frac{\lambda}{2\sin\alpha}$
C) $\frac{\lambda}{4\sin\alpha}$ D) $\frac{\lambda}{4\cos\alpha}$

8) Two blocks A and B of masses m_A = 1 kg and m_B = 3 kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is (Take g = 10 m/s²)



A) 16 N B) 40 N
C) 12 N D) 8 N

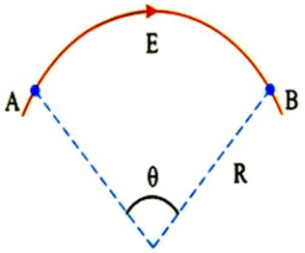
9) Figure shows an electric line of force which curves along a circular arc. The magnitude of electric field intensity is same at all points on this curve and is equal to E. If the potential at A is V, then the potential at B is

A) V - ERθ

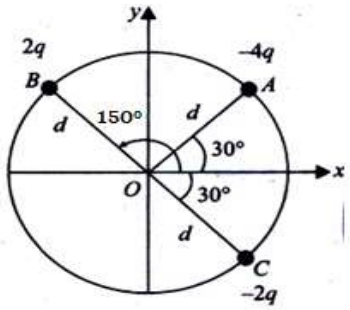
B) V – E2R sin $\frac{\theta}{2}$

C) V + ERθ

D) V+2ER sin $\frac{\theta}{2}$



10) Three charged particles A, B and C with charges -4q, 2q and -2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is



A) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ B) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

C) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ D) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

11) A disc of mass in lies flat on a smooth horizontal table. A light string runs halfway around it as shown in figure. One end of the string is attached to a particle of mass m and the other end is being pulled with a force F. There is no friction between the disc and the string. Find acceleration of end of the string to which force is being applied.

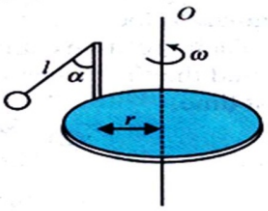
A) $\frac{2F}{m}$ B) $\frac{3F}{m}$

C) $\frac{F}{m}$ D) $\frac{5F}{m}$

12) A bar magnet with a magnetic moment 5.0 Am^2 is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet form parallel to antiparallel position relative to the field direction is_____.

A) 4 J B) 1 J
C) 2 J D) Zero

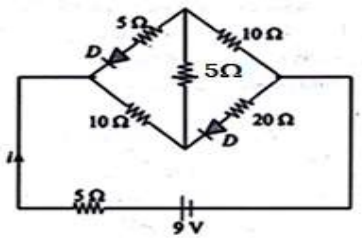
13) A ball of mass m is attached to the end of a thread fastened to the top of a vertical rod which is fitted to a horizontally revolving round table as shown. If the thread forms an angle α with the vertical, the angular velocity ω of table is



A) $\sqrt{\frac{g}{l\cos\alpha}}$ B) $\sqrt{\frac{r}{g\tan\alpha}}$

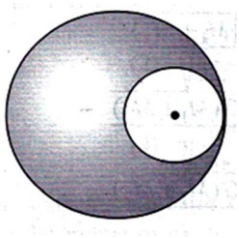
C) $\sqrt{\frac{g\tan\alpha}{r+l\sin\alpha}}$ D) $\sqrt{\frac{g\tan\alpha}{r}}$

14) The current i in the network is



A) 0.3 A B) 0.2 A
C) 0.6 A D) 0 A

15) From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is (G = gravitational constant)



A) $\frac{-GM}{2R}$ B) $\frac{-GM}{R}$

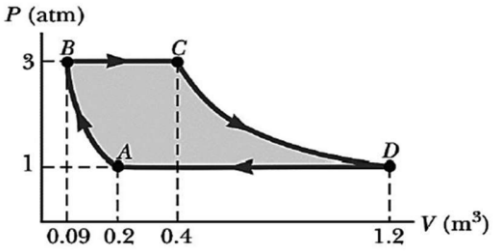
C) $\frac{-2GM}{3R}$ D) $\frac{-2GM}{R}$

16) In a hydrogen like atom an electron is orbiting in an orbit having quantum number n. In time 7 nano - second the electrons jumps back to orbit having quantum number $\frac{n}{2}$. τ be the average torque acted on the electron during the above process, then find $\frac{\tau}{5}\times 10^{27}$ in N – m.

(Given : $\frac{h}{\pi}=2.1\times 10^{-34} J-s$) (n = 4)

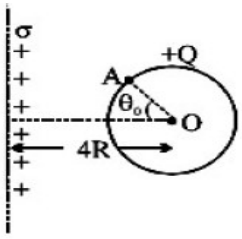
A) 3 B) 4 C) 5 D) 6

17) A sample of an ideal gas goes through the process shown in Figure. From B to C, it is isobaric with 372 kJ of energy entering the system by heat; Take 1 atm = 1.00 × 10^5 Pa. The gas is



A) monoatomic B) diatomic
C) polyatomic D) Mixture

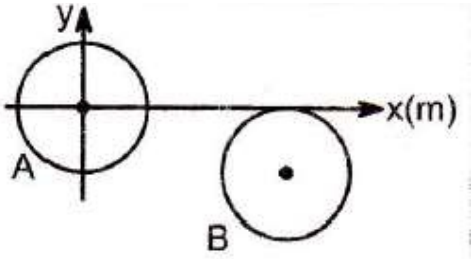
18) A conducting sphere of radius R and charge Q is placed near a uniformly charged nonconducting infinitely large thin plate having surface charge density σ. Then find the potential at point A(on the surface of sphere) due to charge on sphere (here $K=\frac{1}{4\pi\in_0}, \theta_0=\frac{\pi}{3}$)



A) $K\frac{Q}{R}-\frac{\sigma}{4\in_0}R$ B) $K\frac{Q}{R}-\frac{\sigma R}{\in_0}$

C) $K\frac{Q}{R}$ D) $\frac{KQ}{R}+\frac{\sigma R}{\in_0}$

19) Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,-R,R) respectively , in a coordinate system as shown. Ball A, moving along positive x-axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic.



Impulse of the force exerted by A on B during the collision, is equal to

A) $\left(\sqrt{3}m\hat{i}+3m\hat{j}\right)kg-m/s$

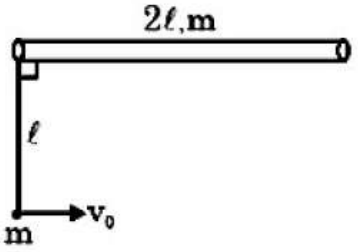
B) $\left(\frac{\sqrt{3}}{2}m\hat{i}-3m\hat{j}\right)kg-m/s$

C) $\left(3m\hat{i}-\sqrt{3}m\hat{j}\right)kg-m/s$

D) $\left(2\sqrt{3}mi+3m\hat{j}\right)kg-m/s$

SUBJECT : CHEMISTRY

20) A uniform rod of mass m, length 2ℓ lies on smooth horizontal surface. A particle of mass m is connected to a light, inextensible string of length ℓ whose other end is connected to rod. Initially string is just taut and string and rod are at perpendicular. If particle given velocity v_0 perpendicular to string. Then immediately after tension in the string is : (Entire arrangement is on the smooth horizontal surface)



A) $\frac{mv_0^2}{l}$ B) $\frac{mv_0^2}{2l}$

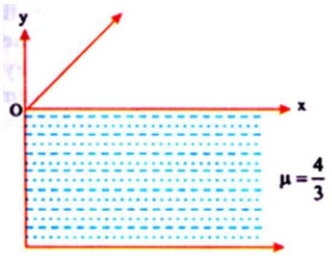
C) $\frac{mv_0^2}{5l}$ D) $\frac{mv_0^2}{4l}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

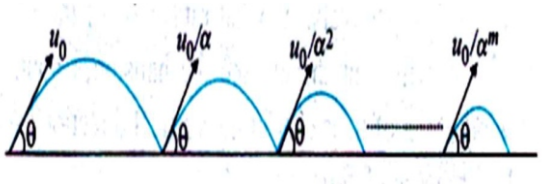
This section contains 05 questions. The answer to each question is a Numerical value. If the Answer in the decimals, Mark nearest Integer only question will be evaluated according to the following marking scheme:

Marking scheme: +4 for correct answer, -1 in all other cases.

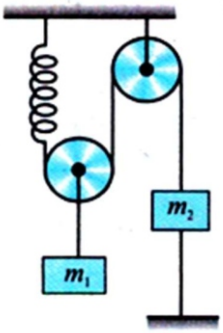
21) The bottom of a water tank is lined with a plane mirror. A particle is projected from origin '0' under gravity. The equation of trajectory is $y = a(x-x^2)$. The depth of tank is 4a and this is filled completely. The maximum separation between the particle and its image viewed from outside water is ____ $\left(\text{Take } a = \frac{6}{13}\right)$ (round of the answer to the nearest integer)



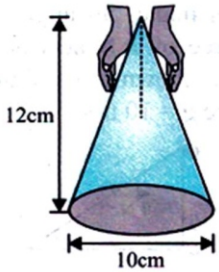
22) A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of $\frac{u_0}{\alpha}$. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is_____ .



23) In figure shown, pulleys are ideal. Initially the system is in equilibrium and string connecting m_2 to rigid support below is cut. What is the initial acceleration (in m/s^2) of m_2? (Given m_1 = 9kg; m_2 = 3 kg)



24) With two hands, you hold a cone motionless upside down, as shown in Fig. The mass of the cone is m = 1 kg, and the coefficient of static friction between you fingers and the cone is μ = 0.5. What is the minimum normal force (in N) you must apply with each hand in order to hold up the cone? Consider only translational equilibrium.



25) A magician pulls a tablecloth from one edge from under a 200-g mug located 30.0 cm from the opposite edge of the cloth. The cloth exerts a friction force of 0.100 N on the mug, and the cloth is pulled with a constant acceleration of 3.00 m/s^2. If $\sqrt{K}$ sec is the time taken by the magician to pull the cloth completely out from under the mug what is the value of K?

SECTION – I

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

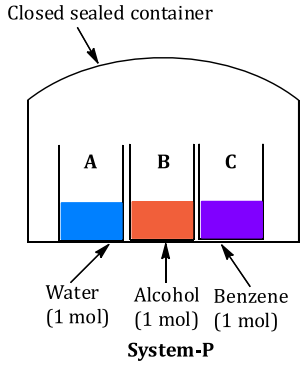
26) Consider the following system: Vapour pressure of pure components at a specific temperature are given as

$P^o_{Water} = 40\,mm\ of\ Hg$

$P^o_{Alcohol} = 60\,mm\ of\ Hg$

$P^o_{Beznene} = 100\,mm\ of\ Hg$

Also, water and alcohol are completely miscible with each other and form ideal solution, while water and benzene and alcohol and benzene are completely immiscible with each other. The following system is taken initially and finally allowed to established equilibrium inside the large sealed container.



System-P

The final pressure in sealed container in system–P will be

A) 60 B) 100

C) 140 D) 150

27) From the following reactions,

$2CoF + F_2 \rightarrow 2CoF_3$

$(CH_2)_n + 4\ n\ CoF_3 \rightarrow (CF_2)_n + 2n\ HF + 4n\ CoF_2$

calculate how much F_2 will be consumed to produce 1 kg of $(CF_2)_n$. (F = 19)?

A) 1.52 kg B) 2.04 kg

C) 0.76 kg D) 4.56 kg

28) 14g of an alkene C_nH_{2n} and excess of $O_2(g)$ at 273K and 2 atm pressure are exploded in a 22.4L of a steel container. After the complete combustion of alkene, the temperature is raised to 546K and pressure is 4.5 atm. The alkene is

A) C_2H_4 B) C_3H_6

C) C_4H_8 D) C_5H_{10}

29) The heat of combustion of liquid CH_4 (methane) is –800 kJ/mol and that of liquid C_3H_8 (propane) is -2156 kJ/mol. Both require liquid O_2 for their combustion.

ISRO (Indian Space and Research Organization) want to use one of these fuels for their rocket motors in Chandrayan–4 mission. Keeping remember that rockets are weight sensitive i.e., lesser the weight, better the rocket fuel, which fuel should ISRO use?

A) Methane B) Propane

C) 1:3 by mole mixture of Methane and Propane

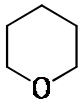
D) 3:1 by mole mixture of Methane and Propane

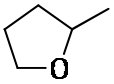
30) A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_o, in which an ideal gas is contained under the same pressure P_o and at the same temperature. What work has to be performed in order to increase the volume of one part of gas isothermally η times when compared to that of the other by slowly moving the piston?

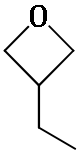
A) $P_oV_o \ln \eta$ B) $P_oV_o \ln\frac{(\eta+1)^2}{4\eta}$

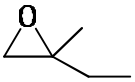
C) $P_oV_o \ln\frac{(\eta+1)}{2}$ D) $2P_oV_o \ln \eta$

31) An ether X with molecular formula $C_5H_{10}O$ reacts with excess of hot aq. HI to give a product which on further reaction with hot NaOH in ethanol forms 1,3-pentadiene. Structure of X is

A) 

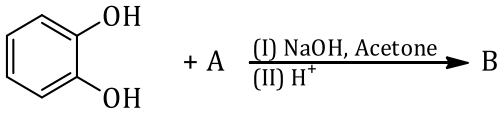
B) 

C) 

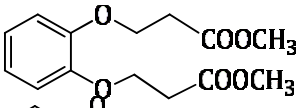
D) 

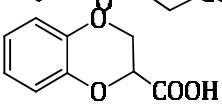
32) Consider the following sequence of reaction

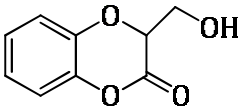
$$H_2C=CH-COOCH_3 + Br_2 \xrightarrow{CCl_4} A$$

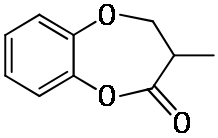


The end product (B) is:

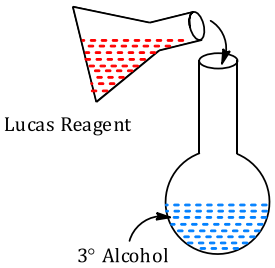
A) 

B) 

C) 

D) 

33) What observation would you get in the following?



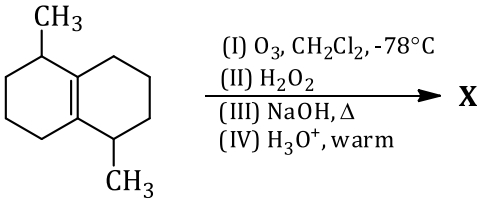
A) White Turbidity of alkyl chloride will be formed immediately

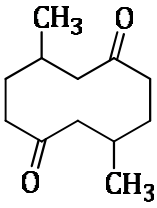
B) White Turbidity of alkyl chloride will be formed in 3–5 minute

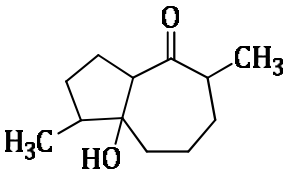
C) White Turbidity of alkyl chloride will be formed in 5–10 minute on heating

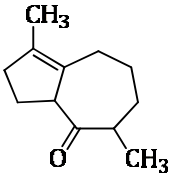
D) No visible observation will be seen

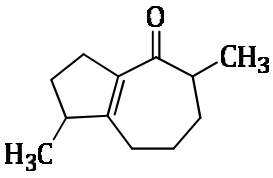
34) The product X of the following sequence of reaction is



A) 

B) 

C) 

D) 

35) Arrange the following cyclic compounds $(CH_2)_nCHY$ in their increasing order of % SYN elimination.

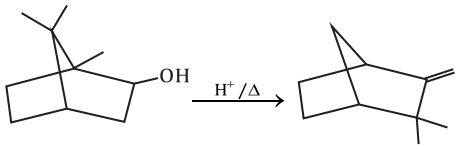
A) Cycloheptyl < Cyclohexyl < Cyclopentyl < Cyclobutyl

B) Cyclohexyl < Cycloheptyl < Cyclopentyl < Cyclobutyl

C) Cyclphexyl < Cycloheptyl < Cyclobutyl < Cyclopentyl

D) Cyclobutyl < Cyclopentyl < Cyclohexyl < Cycloheptyl

36) Consider the following conversion.



Consider the following statements:

(I) Methyl shift takes place in the following conversion

(II) 2^{o} carbocation forms

(III) 3^{o} carbocation forms

(IV) Ring expansion takes place

(V) Carbocation at bridged carbon takes place

Which of the above statement(s) is(are) correct

A) II, III only B) I, II and III

C) I, II and IV D) All

37) Given wave function represents which orbital of hydrogen

$$\psi = \frac{1}{81}\sqrt{\frac{2}{\pi}}\left(\frac{1}{a_o}\right)^{3/2}\left(\frac{6r}{a_o}-\left(\frac{r}{a_o}\right)^2\right)\times e^{-r/3a_o}\cos\theta$$

(where θ = angle from z axis)

A) $3p_x$ B) $3d_{yz}$

C) $3p_z$ D) $3d_{xy}$

38) In the structure of H_2CSF_4, to decide the plane in which C = S is present the following bond angle value are given

Axial FSF angle (idealised = 180°) → 170°

Equatorial FSF angle (idealised = 120°)→ 97^{o}

After deciding the plan of double bond, which of the following statements is CORRECT?

(I) Two C–H bonds are in the same plane of axial S–F bonds.

(II) Two C–H bonds are in the same plane of equatorial S–F bonds.

(III) Maximum six atoms are in the same plane.

(IV) Equatorial S–F bonds are perpendicular to plane of π bond.

The correct statement is/are

A) I only B) I and III Only

C) II and III Only D) All

39) Two equilibria simultaneously exist in a vessel at 25°C

$NO_{(g)} + NO_{2(g)} \rightleftharpoons N_2O_{3(g)}$; K_{p1} (say)

$2NO_{2(g)} \rightleftharpoons N_2O_{4(g)}$ $K_{p2} = 8\ atm^{-1}$

If initially only NO and NO_2 are present in a 3:5 mole ratio and the total pressure at equilibrium is 5.5 atm with the pressure of NO_2 at 0.5 atm, the value of $K_{P1}(atm^{-1})$ is

A) 0.4 B) 0.6

C) 1.0 D) 1.2

40) The indicator constant (K_{In}) of phenolphthalein (8.3 (colourless)–10.3 (Pink)) is approximately 1.0×10^{-10}. A solution is prepared by adding 101.0 mL of 0.01 M sodium hydroxide to 100.0 mL of 0.01M hydrochloric acid. If a few drops of phenolphthalein are now added, what fraction of the indicator is converted to its Pink form?

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{1}$

41) Consider the following statements

(I) Large rings with more than six carbon atoms are stable but difficult to prepare.

(II) Decreasing order of thermal stability of cyclic rings is 6 > (7, 5) > (8, 9) > 4 > 3.

(III) Decreasing order of probability of ring closure is 3 > 4 > 5 > 6 > 7 > 8 > 9

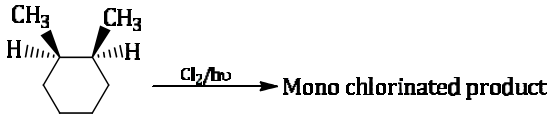
(IV) Ease of synthesis of cyclo compounds is 5 > (3, 6) > (4, 7, 8, 9)

Which of the above statements is/are correct?

A) I and II B) II and III

C) III and IV D) All

42) Consider the following



(I) Four 3° monochloro product will be obtained

(II) Seven fractions are obtained on fraction distillation

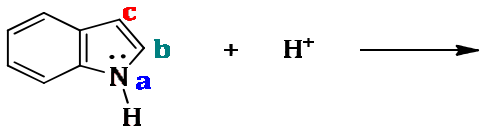
(III) Eight 2° monochlorinated product will be obtained

The correct statement is/are

A) I only B) I and III Only

C) II and III Only D) All

43) Protonation on indole occurs at

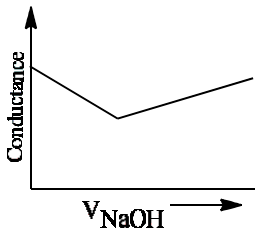


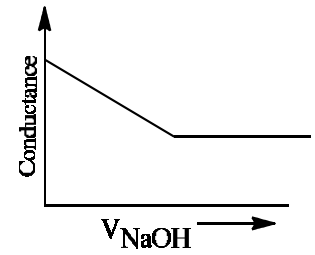
Indole

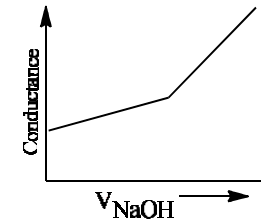
A) a B) b

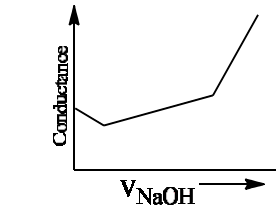
C) c D) None of these

44) CH_3COOH is titrated with NaOH(aq) conductometrically, Which of the following give the best graphically representation of the titration :

A)


B)


C)


D)


45) To check the principle of multiple proportions, a series of pure binary compounds (PQ) were analyzed and their composition is tabulated below. The correct option(s) is (are)

Compound	Weight % of P	Weight % of Q
1	50	50
2	44.4	55.6
3	40	60

(I) If empirical formula of compound 3 is P_3Q_4, then the empirical formula of compound 2 is P_3Q_5.

(II) If empirical formula of compound 3 is P_3Q_2 and atomic weight of element P is 20, then the atomic weight of Q is 45.

(III) If empirical formula of compound 2 is PQ, then the empirical formula of the compound 1 is P_5Q_4

(IV) If atomic weight of P and Q are 70 and 35, respectively, then the empirical formula of compound 1 is P_2Q.

The correct statement is/are

A) I only B) I and III Only

C) II and III Only D) All

SECTION-II

(NUMERICAL VALUE ANSWER TYPE)

This section contains 05 questions. The answer to each question is a Numerical value. If the Answer in the decimals, Mark nearest Integer only question will be evaluated according to the following marking scheme:

Marking scheme: +4 for correct answer, -1 in all other cases.

46) Acetic acid has $K_a = 2.0\times10^{-5}$ while formic acid has $K_a = 3.0\times10^{-4}$. X is magnitude of the emf of the cell

$$Pt(H_2)|\underbrace{CH_3COOH}_{0.1M}+\underbrace{CH_3COONa}_{0.1M}||\underbrace{HCOOH}_{1.0M}+\underbrace{HCOONa}_{0.1M}|Pt(H_2)$$ at 25°C What will be value of $(100 \times x)$

47) The number geometric isomers possible for the complex

$[Col_2Cl_2]^-$ ($L = H_2NCH_2CH_2O^-$) is

48) Consider the following reversible reaction which is first order in both direction:

$A \underset{k_b}{\overset{k_f}{\rightleftharpoons}} P$; ΔH = 23.0kJ/mol and K_C = 4 at 300 K

Also the following Arrhenious equation applies appropriately:

$$\log k_f(min^{-1}) = 28 - \frac{9600}{T}$$

What is the activation energy (in $kJmol^{-1}$) for the formation of A?

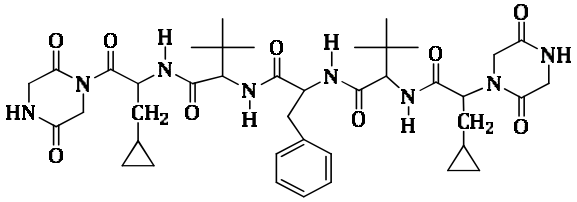
49) The rate law of the reaction is given as 2A + B $\longrightarrow$ Product

Rate = $k[A]^2[B]$

$[A]_{initial}$	$[B]_{initial}$	$t_{1/2}$
5×10^{-7} M	3.0×10^{-4} M	400
10×10^{-7} M	3.0×10^{-4} M	400/x

The value of x is

50) Consider the following peptide



How many total number of different amino acid will be obtained on acidic hydrolysis of the peptide

SUBJECT : MATHEMATICS

SECTION – I

(SINGLE CORRECT ANSWER TYPE)

This section contains 15 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51) Let P(x) be a polynomial of degree less than 100 such that P(1), P(–1) and P(0) are distinct non-zero quantities. If $\frac{d^{100}}{dx^{100}}\left(\frac{P(x)}{x^3-x}\right)=\frac{f(x)}{g(x)}$ for some polynomials f(x) and g(x) then the value of $\frac{n}{4}$ is _____ (where 'n' is the smallest possible degree of f(x))

A) 50 B) 5.25

C) 51 D) 52

52) z_1, z_2, z_3 are three unimodular complex numbers which are also roots of the equation $z^3 + az^2 + bz + 1 = 0$ (where a, b are complex numbers). If tangents are drawn to $|z| = 1$ at points z_1, z_2, z_3 which intersect pair wise at points ω_1, ω_2, ω_3 are roots of the equation.

A) $(ab - 1) z^3 + 2 (b^2 - a) z^2 + 8bz + 8 = 0$

B) $(ab - 1) z^3 + 2 (b^2 + a)z^2 - 8bz + 8 = 0$

C) $(ab - 1) z^3 - 2(b^2 + a)z^2 + 8bz + 8 = 0$

D) $(ab - 1)z^3 + 2 (b^2 + a)z^2 + 8bz + 8 = 0$

53) The complex numbers z_1, z_2, z_3 are the vertices of triangle ABC such that $|z_1-1|=|z_2-1|=|z_3-1|$ and $\arg\left(\frac{z_3-z_1}{z_2-z_1}\right)=\frac{\pi}{6}$ then the value of $(z_2(z_2-1)-z_3(z_2+1)+(z_3+1)(z_3-1))=.....$

A) 2 B) 3

C) – 2 D) 5

54) The number of functions from the set A = {0, 1, 2} into the set B = {0,1,2,3,4,5,6,7} such that f(i) ≤ f(j) for i < f and i, j ∈ A is

A) ${}^8C_3$ B) ${}^8C_3+2({}^8C_2)$

C) ${}^{10}C_3$ D) ${}^9C_3$

55) A pair of numbers is picked up, randomly (without replacement) from the set {1, 2, 3,5, 7, 11, 12, 13, 17, 19}. The probability that the number 11 was picked given that the sum of the numbers was even is nearly

A) 0.1 B) 0.125

C) 0.24 D)0.18

56) Let a point R lies on the plane x – y + z - 3 = 0 and P be the point (1, 1, 1). A point Q lies on PR such that $PQ^2 + PR^2 = k$ ($\neq 0$) then the equation of locus of Q is

A) $\left[(x-1)^2+(y-1)^2+(z-1)^2\right]\left[1+\frac{4}{(x-y+z-1)^2}\right]=k$

B) $\left[(x-1)^2+(y-1)^2+(z-1)^2\right]\left[1-\frac{4}{(x-y+z-1)^2}\right]=k$

C) $\left[(x-1)^2+(y-1)^2+(z-1)^2\right]\left[1-\frac{4}{(x-y+z-1)^2}\right]=k$

D) $\frac{1}{(x-1)^2}+\frac{1}{(y-1)^2}+\frac{1}{(z-1)^2}+\frac{(x-y+z-1)^2}{4}=k$

57) Let $I_n = \int_0^1 \frac{x^n}{x^{2019}-1}dx$ and $J_n = \int_0^1 \frac{x^n}{x^{2019}+1}dx$, where n ∈ N. Matrices A = $[a_{ij}]_{3\times 3}$, and B $[b_{ij}]_{3\times 3}$, are defined as $a_{ij} = \begin{cases} I_{2019+i} - I_i & , \ i = j \\ 0 & , \ i \neq j \end{cases}$ and $b_{ij} = \begin{cases} J_{2020+j} + J_{i+1} & , \ i \neq j \\ 0 & , \ i \neq j \end{cases}$ then the value of $\sqrt{|A^{-1}| + tr(B^{-1})} =$ (Here || represents determinant of a matrix and tr (A) represents trace a of matrix A^{-1} represents inverse of a matrix)

A) 6 B) 5 C) 4 D) 3

58) Let OABCD be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel. Also OA : CB = 2 : 1 and OD : . AB = 1 : 3. Let X be the point of intersection of the diagonals OC and AD. If the ratio $\frac{OX}{XC}$ is given by $\frac{p}{q}$, (where p, q are positive integers which are relatively prime to each other), then p + q equals,

A) 6 B) 7
C) 8 D) 9

59) Let $f(t) = |t| + |t-1| \forall t \in R$ and $g(x) = \begin{cases} \max.(f(t)), & x-1 \le t \le x, \quad 0 \le x \le 1 \\ 3-x, & 1 < x \le 2 \end{cases}$ the number of points where g(x) is non-derivable in [0,2] is

A) 5 B) 1
C) 15 D) 6

60) If y is implicit differentiable function of x such that $y(x+y)^2 = x$ *then* $\int \frac{dx}{x+3y} =$

A) $ln\left((x+y)^2+1\right)+c$

B) $\frac{1}{2} ln\left((x-y)^2+1\right)+c$

C) $ln\left((x-y)^2+1\right)+c$

D) $\frac{1}{2} ln\left((x+y)^2+1\right)+c$

61) The sequence of matrices in defined as given below m(1) = [1]

$m(2) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, m(3) = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and so on

then the trace of m(17) is

A) 27,895 B) 23, 679
C) 27,897 D) 27, 987

62) P, Q, and R are the feet of the normal drawn to a parabola $(y-3)^2 = 8(x-2)$. A circle cuts the above parabola at points P, Q, R and S. Then this circle always passes through the point.

A) (2,3) B) (3,2)
C) (0,3) D) (2,0)

63) A and B are two non-singular matrices such that $A^6 = I$ and $AB^2 = BA$ ($B \neq I$). A value of k so that $B^k = I$ is

A) 31 B) 32
C) 64 D) 63

64) Circle are drawn on chords of the rectangular hyperbola xy = 4 parallel to the line y = x as diameters. All such circles passes through two fixed points whose one of the co-ordinate is

A) (2,3) B) (2,–2)
C) (–2,2) D) (–2,–2)

65) ABCD is a square of unit side. It is folded along the diagonal AC, So that the plane ABC is Perpendicular to the plane ACD. The shortest distance between AB and CD is

A) $\sqrt{3}$ B) $\frac{1}{\sqrt{3}}$

C) $\frac{\sqrt{3}}{\sqrt{2}}$ D) $\frac{\sqrt{2}}{\sqrt{3}}$

66) Mean of the numbers $\frac{50_{C_0}}{1}, \frac{50_{C_2}}{3}, \frac{50_{C_4}}{5}, \ldots\ldots \frac{50_{C_{50}}}{51}$ is

A) $\frac{2^{49}}{(39)(17)}$ B) $\frac{2^{50}}{51}$

C) $\frac{2^{49}}{51}$ D) $\frac{2^{49}}{54}$

67) 4-points whose position Vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a}+(2\sin 2\beta)\vec{b}+(3\sin 3\gamma)\vec{c}-\vec{d}=\vec{0}$ then the least value of $\sin^2\alpha+\sin^2 2\beta+\sin^2 3\gamma$ is

A) $\frac{1}{14}$ B) 14

C) 6 D) $\frac{1}{\sqrt{6}}$

68) The lines $y = m_1x$, $y = m_2x$ and $y = m_3x$, make equal intercepts on the line $x + y = 1$. Then

A) $2(1 + m_1)(1 + m_3) = (1 + m_2)(2 + m_1 + m_3)$ B) $(1 + m_1)(1 + m_3) = (1 + m_2)(1 + m_1 + m_3)$

C) $(1 + m_1)(1 + m_2) = (1 + m_3)(2 + m_1 + m_3)$

D) $2(1 + m_1)(1 + m_3) = (1 + m_2)(1 + m_1 + m_3)$

69) For a twice differentiable function f(x), g(x) is defined as $g(x) = f'(x)^2 + f''(x) f(x)$ on [a,e]. If for $a < b < c < d < e$, $f(a) = 0$, $f(b) = 2$, $f(c) = -1$, $f(d) = 2$, $f(e) = 0$ then the minimum number of zeros of g(x) is

A) 6 B) 7

C) 8 D) 4

70) The value of $\frac{1+\sin 22^0 \sin 33^0 \sin 35^0}{\cos^2 22^0+\cos^2 33^0+\cos^2 35^0}$ is _____.

A) 0.3 B) 0.5

C) 0.4 D) 0.6

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)
This section contains 05 questions. The answer to each question is a Numerical value. If the Answer in the decimals, Mark nearest Integer only question will be evaluated according to the following marking scheme:
Marking scheme: +4 for correct answer, -1 in all other cases.

71) Let $a_n = \sum \frac{\left({}^nC_r\right)^2\left((r+2)^2-(n-r)^2\right)}{(r+1)^2(r+2)^2}$. If $\prod_{k=1}^{2025} a_k = \frac{p}{q}$, where p,q are co-prime positive integers, then the value of $|p - q|$ =

72) If the real numbers α, β satisfy the equation $\alpha^3 - 3\alpha^2 + 5\alpha - 17 = 0$; $\beta^3 - 3\beta^2 + 5\beta + 11 = 0$ then $(\alpha + \beta)$ is equal to

73) If $f(x) = \frac{x-a}{bx^2+cx+2}$, where a,b,c $\in$ R, f(–1) = 0 and y = 1 is an asymptote of y = f(x) and y = $f^{-1}(x)$ is inverse function of f(x), then area bounded between asymptotes of curve f(x) and $f^{-1}(x)$ is

74) Suppose f(x) and g(x) are differentiable functions such that g(f(x)). f'(g(x)).g'(x) = f(g(x)).g'(f(x)).f'(x) for all real x and $\int_0^a f(g(x))dx = 1 - \frac{e^{-2a}}{2}$ for all real 'a'. Given that g (f(0)) = 1. If the value of g(f (4)) = e^{-k^2}, (where k ∈ N) then is equal to

75) The product of real values of x such that $(\log_x 2)^2 - (\log_2 x)^2 = \log_{2x}\left(\frac{2}{x}\right)$ is is $\frac{1}{\sqrt{x}}$. What is the value of x?